Subatomic Physics

- particle is a countable entity with fixed properties (mass, charge, ...)
  ↳ only some have size (composite) x fundamental
  proton, nucleus, atom — electrons, quarks, ν
- particles behave as waves with $\lambda = \frac{h}{p}$ ⇒ can interfere
  ⇒ we can probe the size of particles by collisions with other
  - at larger distances / low energies: ≈ point-like
  - in free space behave like plane waves (simple)

Special Relativity

- $V_{particle} > 0.1c$ ⇒ cannot neglect SR at high speeds.
- in PP • our frame = ~~#~~ ultimately stationary reference frame
- applications: time dilation in particle decay time
  $E \sim p \sim m$

Lorentz transformation

- frame S and moving S' with coordinates x'

Lorentz factor $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\beta^2}} \in [1, \infty)$

- time and space are not independent (vs. Galilean)!

$$\left.\begin{aligned} t' &= \gamma\left(t - \frac{vx}{c^2}\right) \\ x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \end{aligned}\right\} \text{for } \vec{v} \text{ in } \hat{x}\text{-direction}$$

use transverse quantities to ~~avoid~~ get Lorentz invariant quantities

cross-section

Time dilation: $\boxed{t_{lab} = \gamma\tau}$ ↳ undilated time (in particle's frame)

$\boxed{E = \gamma mc^2}$

☆ μ hitting atmosphere at 30km altitude with $\tau = 2.2\,\mu s$ (lifetime)
$x \cong ct \cong 700m$ ☹ don't reach anywhere near us.
But many still hit the surface of Earth!
$E = 4\,GeV$, $m = 106\,MeV\,[c^2]$ ⇒ $\gamma = \frac{E}{m} \cong 40$ ($v = 0.99965c$)
⇒ lifetime from our perspective: $t = \gamma\tau = 88\,\mu s$ ⇒ $x = \gamma c\tau \cong$ 26km ☺

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Einarbeitung ?

Energy $E = mc^2$

- use Δm between two states to compute released energy

$E^2 = m^2c^4 + p^2c^2 \rightsquigarrow \pm$ solutions?!

↳ kinetic energy : $E_{kin} = E - mc^2 = (\gamma - 1)mc^2$

mass $\Rightarrow \boxed{p = \gamma m v}$

- m is Lorentz invariant (invariant in any frame)
  ↳ common

Four vectors

- space + time $\Rightarrow$ spacetime x^μ
- Energy + momentum $\Rightarrow$ energy-momentum p^μ

$$p_\mu p^\mu = (E\ p_x\ p_y\ p_z)\begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix} = E^2 - (\vec{p}\cdot\vec{p})c^2 = m^2c^4$$

$\Rightarrow$ Lorentz invariant mass

$\Rightarrow$ relativistic energy and momentum in each direction are still conserved

Invariant mass $\Rightarrow$ used often : many particles (e.g. Higgs) decay too fast to reach detector even at large speeds

$H \Rightarrow \underbrace{\mu^+\mu^-}_{Z}\underbrace{\mu^+\mu^-}_{Z}$ $\Rightarrow$ we detect μ's

compute whether Higgs present:

$$m^2c^4 = \begin{pmatrix} E_{4\mu} \\ p_{x,4\mu} \\ p_{y4\mu} \\ p_{z,4\mu} \end{pmatrix}^2 = \begin{pmatrix} \Sigma E_{\mu_i} \\ \Sigma p_{x,\mu_i} \\ \vdots \end{pmatrix}^2 \stackrel{?}{=} m_H^2c^4$$

Natural units

$\hbar \equiv 1$

$c \equiv 1$ $\Rightarrow$ all units defined relative to energy

$1s = 6.58 \times 10^{-25}\,GeV^{-1}$ — express everything as energy equivalent (incl. final result)

$1m = 0.197 \times 10^{-15}\,GeV^{-1}$ — convert back to SI (using direct conversion)

reinsert factors of $\hbar, c$ as required (in the final answer)

decay rate ~ width

$\Gamma = \frac{1}{\tau}$

↳ uncertainty in mass of particle $(\Delta E \Delta t \geq \frac{\hbar}{2})$

SR + QM: Dirac Equation - linear but $E^2 = m^2c^4 + p^2c^2$

$$i\hbar \frac{\partial \psi}{\partial t} = [c\,\alpha \cdot \hat{p} + \beta m c^2]\,\psi$$ (for $s = 1/2$)

matrices to make it work

$\therefore$ ± solutions?!

Dirac didn't throw ⊖ E away ⇒ instead proposed that there are particles with same E but opposite quantum numbers ⇒ antiparticles

correctly predicts magnetic moment of electron, $g_e = 2$ vs. classically $g_e = 1$

vs. Klein-Gordon : $-\frac{\partial^2 \psi}{\partial t^2} = -\nabla^2 \psi + m^2 \psi$

↳ so satisfy both: $\alpha_i^2 = 1$, $\beta = 1$, $\alpha_i\beta + \beta\alpha = 0$ } needed 4x4 matrices
↳ 2 spin states (↑, ↓)
↳ 2 matter states (particle/anti)
⇓
antiparticle ~ particle with negative energy ⇒ going back in time

⇒ antiparticles are necessary for relativistic QM theory

Particle interactions

- particles encounter each other and modify each other's states.
- governed by exchange of force/interaction particles
- which particles can be exchanged is determined by charges
- consequences associated defined in Lagrangian
- elastic interactions $\pi^- + p^+ \to \pi^- + p^+$ - states change but not the fundamental properties
- inelastic interactions - different particles produced

$\pi^- + p^+ \to \pi^- + p^+ + \pi^+ + \pi^-$

$p + e^- \to n + \nu_e$

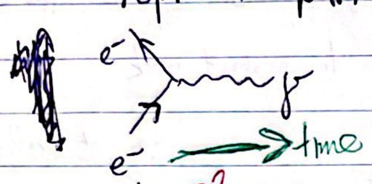
↳ quantum numbers conserved always { charge, lepton number, baryon number

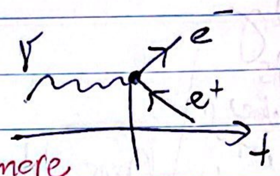
in strong & elmag { parity, charge conjugation

Interaction diagrams

- represent particle interactions → used to calculate interaction rates

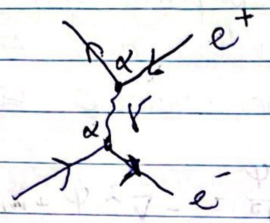
$\alpha \cong \frac{1}{137}$





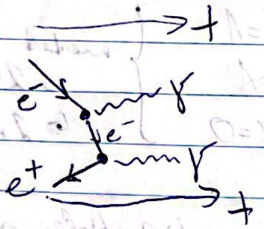
↳ always 2 vertices or more

vertices
↳ coupling of particles with certain coupling strength
↳ at least 2



- antimatter backwards in time
- momentum is conserved at vertices ⇒ single vertex diagram does not conserve energy
- all real interactions need at least 2 vertices, including exchange of virtual particles $\Delta E \Delta t \gtrsim \frac{\hbar}{2}$

$\boxed{e^+e^- \to \gamma\gamma}$



- to calculate a transition, need to consider all possible diagrams with same in and outgoing particles
- consider effect of force particle with mass M_x violation of energy conservation of timescale

$\tau \approx \frac{\hbar}{\Delta E} = \frac{\hbar}{M_x c^2}$

⇒ associated range of force: $r \approx \frac{\hbar}{M_x c}$

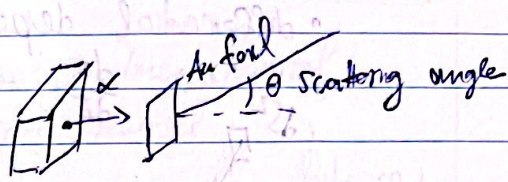
☆ For weak interaction, $M_W = 80\,GeV$, $M_Z = 91\,GeV$

⇒ $r \approx 0.002\,fm$

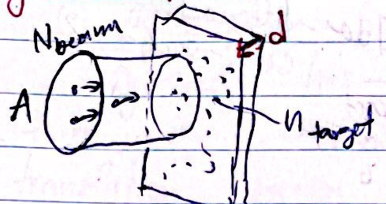
⇒ weak

✶ Scattering : Rutherford

- first scattering experiment



Scattering description



- Number of scattered particles:

$$\boxed{N_{int} = N_{beam}\, n_{target}\, d \cdot \sigma = \frac{N_{beam}\, N_{target}}{A}\, \sigma}$$

N_{int}: # interacting particles (# interactions)
N_{beam}: total number of particles passing through
n_{target}: density of atoms per volume $\frac{N_{target}}{V}$
σ: cross-section

- within experiments: $\boxed{\frac{dN_{int}}{dt} = \Phi\, N_{target}\, \sigma}$

flux $\boxed{\Phi = \frac{1}{A}\frac{dN_{beam}}{dt}}$ — # particles in area A passing in time t

Cross-section

- likelihood of hitting target ∝ "surface"
- for elastic collisions of billiard balls, 'surface' is the geometric area $\sigma = \pi r^2$
- for quantum collisions, cross-section σ is not related to particle size, instead from probability of certain quantum processes occurring, given ~~current~~ density of incoming particles
- not a property of single particle – but processes
  - ↳ depends on e.g. energy, interaction type
  - ↳ $e^- n^0 : \sigma = 10^{-30}\,cm^2$
    $e^- \nu : \sigma - 10^{-42}\,cm^2$
- often given in barn $= 10^{-28}\,m^2$

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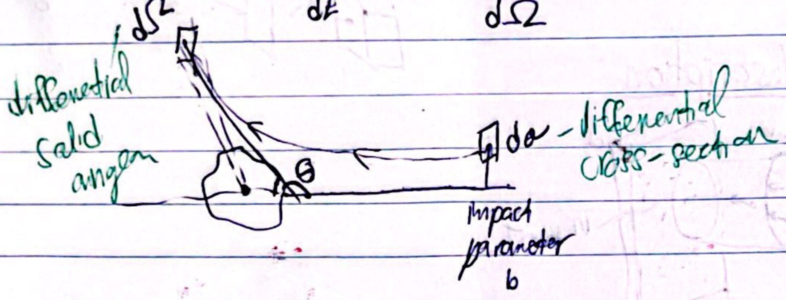
Differential cross-section

- differential dependence on some variable;

  e.g. $\frac{d\sigma}{dE}$ or $\frac{d\sigma}{d\Omega}$



Total / partial cross-section

→ integrate out all dependences, e.g. $\sigma = \int \frac{d\sigma}{d\Omega} d\Omega$

\+ add up all possible subprocesses : $\sigma = \Sigma \sigma_i$

e.g. elastic + inelastic cross-section

Cross-section & luminosity

→ Lorentz invariant : effective surface perpendicular to direction of movement

- express rate of collisions as $\boxed{\frac{dN}{dt} = \mathcal{L}\sigma}$

quality of collider ← luminosity of experiment (things dependent on experiment)

vs. σ : physically intrinsic

Symmetries

- = properties that are invariant under some transformation
  - ✲ translation, rotation
  - ✲ discrete: charge conj., parity, time reversal
  - ✲ internal symmetries "space" reversal
    - ↳ ⇒ interaction

Noether's ~~symmetry~~ theorem

- if a system has a continuous symmetry property, there are corresponding quantities conserved in time
  - ✲ translation symmetry ⇒ momentum conservation
  - ✲ rotation sym. ⇒ angular momentum –"–
  - ✲ time sym. ⇒ energy –"–
- formally from Lagrangians:

$\mathcal{L}(q, \dot{q})$: from principle of least action:

position – velocity

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) = \frac{\partial \mathcal{L}}{\partial q}$$

for free particles: $\mathcal{L} = E_{kin} = \frac{1}{2} m (\dot{q})^2$

Translation: $q \to q + \Delta q$

$$\dot{q} \to \frac{\partial}{\partial t}(q + \Delta q) = \dot{q} + \cancel{\frac{\partial \Delta q}{\partial t}} \to 0$$: assume constant translation

But $\mathcal{L}$ only depends on $\dot{q}$ ⇒ $\frac{\partial \mathcal{L}}{\partial q} = 0$

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) = \frac{d}{dt} m\dot{q} = 0$$

$$\Rightarrow \boxed{\underbrace{m\dot{q}}_{= p} = \text{const.}}$$ conservation of momentum

Quantum? ⇒ use Hamiltonian instead of Lagrangian.

⇒ if Hamiltonian is invariant wrt certain transformation, we can associate conserved quantity

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi$$

$$H\Psi = E\Psi$$

H invariant under translation

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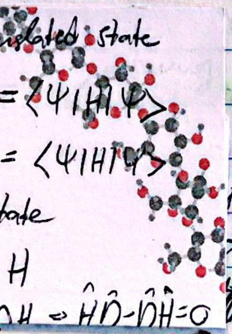

↙ translated state

$$\langle \Psi_T | H | \Psi_T \rangle = \langle \Psi | H | \Psi \rangle$$

$$\langle \Psi | D^{-1} H D | \Psi \rangle = \langle \Psi | H | \Psi \rangle$$

for arbitrary state

$$\Rightarrow D^{-1} H D = H$$

$$HD = DH \Rightarrow \hat{H}\hat{D} - \hat{D}\hat{H} = 0$$

Translation: H(…

- define translat…

$\hat{p} := -i$…

1st order Taylor expansion

$$\ldots(\vec{r} + \Delta \vec{r}) \approx \Psi(\vec{r}) + (\Delta\vec{r})\, \vec{\nabla}\Psi(\vec{r})$$

$$\ldots(\vec{r}) + \Delta\vec{r}\, \frac{i}{\hbar} \hat{p}\, \Psi(\vec{r})$$

$$\ldots \left[1 + \Delta\vec{r}\, \frac{i}{\hbar} \hat{p}\right] \Psi(\vec{r})$$

$$\Rightarrow D \approx 1 + \Delta\vec{r}\, \frac{i}{\hbar} \hat{p}$$

⇒ Now check $\hat{D}$ commutes with $\hat{H}$ (⇒ conservation)

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Global symmetries

= apply equally to full system
e.g. translation, rotation

- In SM: Poincaré symmetry

Standard model

⇒ Conserved: momentum, angular momentum, energy
⇒ Hamiltonian commutes with all of the associated operators

Angular momentum – recap

- 2 types: intrinsic ($\vec{S}$), orbital ($\vec{L}$)
- fundamental particles only have spin $\vec{S}$
  ↳ point like, half-integer for fermions
- composite particles / particle systems also have orbital angular momentum $\vec{L}$ (integer values)
- total angular momentum $\vec{J} = \vec{L} + \vec{S}$

↳ orbital ang. m. and spin have separate ψ

$\Psi(\vec{r}) = \psi(\vec{r})\,\chi$

$\hat{L}$ only acts on ψ, $\hat{S}$ only acts on χ

- $\vec{J}$ is always conserved
- $\vec{L}, \vec{S}$ often conserved separately but not always

Composite particles

- spin of composite particle P determined in rest frame (spin = angular momentum in rest frame)
  ↳ in rest frame: $\vec{S}_p = \vec{J}$ ↳ total angular momentum of particle system

- for mesons: $\vec{S} = \vec{S}_q + \vec{S}_{\bar{q}} = 0 \vee 1$

scalar mesons: $S = 0$ $(\downarrow\uparrow - \uparrow\downarrow)$

vector mesons: $S = 1$ $(\uparrow\uparrow, \downarrow\downarrow, \uparrow\downarrow + \uparrow\downarrow)$

For $L = 0$: $J = S = L = 0$: ${}^1S_0$; $J = S = 1, L = 0$: ${}^3S_1$

lowest energy state ⇝ $\pi^+, K, D, \ldots$ (K: s, D: c) ; ρ^+, K^*, D^*

For lower energy, low L

- for baryons: $\uparrow\uparrow\uparrow$ or $\uparrow\uparrow\downarrow$

$S_p = J = 3/2$; $1/2$

Discrete symmetries

~~[illegible]~~ • invert a certain property
- parity P $\neq$ rotation
- charge conjugation C
- time reversal T $\neq$ time movement sym.

• applying twice doesn't modify state

☆ parity: $P^2 \psi(\vec{r},t) = P \psi(-\vec{r},t) = \psi(\vec{r},t)$

$\Rightarrow$ eigenvalue $\underset{\text{particle}}{P_a} = \pm 1$

• P, C, T conserved in strong & elmag interactions
• P, C strongly violated in weak interactions
• T weakly violated — " —

P Intrinsic parity = reversal of spatial coordinates

• applied to momentum eigenstate: $\psi_p(\vec{r},t) = e^{i(\vec{p}\cdot\vec{r} - Et)}$

$\hat{P}\, \psi_{p}(\vec{r},t) = P_a\, \psi_p(-\vec{r},t) = P_a\, \psi_{-p}(\vec{r},t)$

not the same

$\Rightarrow$ only particle at rest is eigenstate of $\hat{P}$ with eigenvalue P_a

• P_a = intrinsic parity of particle
• scalar $s \to s$ even — e.g. E energy
• vector $\vec{r} \to -\vec{r}$ odd — e.g. $\vec{E}$ electric field, $\vec{r}, \vec{p}$
• axial vector $\vec{L} \to \vec{L}$ even — e.g. $\vec{B}, \vec{L}, \vec{S}$

$\hookrightarrow \vec{L} = \vec{r}\times\vec{p} \to (-\vec{r})\times(-\vec{p}) = \vec{r}\times\vec{p}$

• if particle has definite orbital angular momentum: ψ_{nlm}

$\hat{P}\, \psi_{nlm}(\vec{r}) = \hat{P}\, \underbrace{R_{nl}(r)}_{\text{invariant under } \hat{P}}\, \underbrace{Y_{lm}^{m}(\theta,\phi)}_{\text{not invariant}}$

$\theta \xrightarrow{\hat{P}} \theta + \pi$

$\phi \to$

☆ $Y_0^0 = \sqrt{\frac{1}{4\pi}} \Rightarrow$ even - $l=0$

☆ $Y_1^0(\theta,\phi) = \sqrt{\frac{3}{4\pi}} \cos\theta \xrightarrow{\hat{P}} \sqrt{\frac{3}{4\pi}} \cos(\theta+\pi) = -\sqrt{\frac{3}{4\pi}} \cos\theta \Rightarrow$ odd, $l=1$

☆ $Y_1^{\pm 1}(\theta,\phi) = \mp\sqrt{\frac{3}{8\pi}} \sin\theta\, e^{\pm i\phi} \xrightarrow{\hat{P}} \mp\sqrt{\frac{3}{8\pi}} \sin\theta\, e^{\pm i\phi} \Rightarrow$ odd, $l=1$

$\boxed{\text{• even/odd values of } l \Rightarrow \text{even/odd under parity}}$

- for matter: particle / antiparticle with $S=1/2$: $P_f P_{\bar{f}} = -1$
  $\Rightarrow$ opposite parities for matter / antimatter
- $e^+ + e^- \to \gamma + \gamma$: $P_{e^+e^-} = P_{\gamma\gamma} = (-1)^{l_\gamma}$
- by convention; parity of matter (e^-, μ^-, τ^-, q) $:= 1$
  antimatter $:= -1$
  $\downarrow$
  cannot be determined experimentally
- parity of photon $P_\gamma = -1$ (derived in the book)

- parity for hadrons
  - mesons : $P_M = P_q P_{\bar{q}} (-1)^L = (-1)^{L+1}$
    $\hookrightarrow$ for ground state: $L=0$: $P_M = -1$
    $\hookrightarrow$ parity doesn't modify spatial but exchanges $q \leftrightarrow \bar{q}$
  - baryons: $P_B = P_q P_q P_q (-1)^L = (-1)^L$
    $\Rightarrow$ for ground state: $L=0$: $P_B = 1$
    $\hookrightarrow$ antibaryons $P_{\bar{B}} = (-1)^3(-1)^L = (-1)^{L+1}$
    $\hookrightarrow$ for ground state $L=0$: $P_{\bar{B}} = -1$
- parity in hadron decay
  ☆ $\pi^0 \Rightarrow \gamma\gamma$
  $-1 = P_{\pi^0} = P_\gamma^2 (-1)^{l_\gamma} = (-1)^{l_\gamma} \Rightarrow l_\gamma = \text{odd}$
  $\hookrightarrow$ validated experimentally

C Charge conjugation

$\hat{C}$:
- replaces particles by antiparticles
  $\Rightarrow$ $Q \to -Q$ but also all quantum numbers inverted (e.g. baryon number becomes opposite)
  magnetic moment — " —
- eigenstates of C = only particles w/o distinct antiparticle e.g. γ, π^0 (not n^0)
  $\hookrightarrow \hat{C}|\alpha, \psi\rangle = C_\alpha |\alpha, \psi\rangle$ $\rightsquigarrow$ must be electrically neutral but also
  $\hookrightarrow$ C-parity
- for other particles: $\hat{C}|b, \psi\rangle = |\bar{b}, \psi\rangle$

$\hookrightarrow$ only eigenstates in pairs : $\hat{C}|a\psi_1, \bar{a}\psi_2\rangle = |\bar{a}\psi_1, a\psi_2\rangle = \pm |a\psi_1, \bar{a}\psi_2\rangle$

$\hookrightarrow$ doesn't change spin ! $\Rightarrow$ newly made antiparticle wouldn't annihilate with the original particle

☆ $\hat{C}|\pi^+\pi^-;L\rangle = (-1)^L|\pi^+\pi^-;L\rangle$

☆ $\hat{C}|f\bar{f};L,S\rangle = (-1)^{L+S}|\bar{f}f;L,S\rangle \Rightarrow \pi^0 (L=S=0) \Rightarrow C_{\pi^0}=1$

validated by $\pi^0 \to \gamma\gamma$

$C_{\pi^0} = C_\gamma^2 = 1$

☆ $C_\gamma = -1$ ~ more in book : "charge which created the photon is flipped with charge conjugation ⇒ also flip for photon"

~~interaction~~ $\vec{E}$ ~~...~~?

$\vec{F}_{Lorentz} = q(\vec{E}+\vec{v}\times\vec{B})$

⇝ -1

T Time reversal : $t \to t' = -t$

$p \to p' = -p$

$L \to L' \to -L$

- ~~...~~ symmetry of strong & elmag but not in weak
- no associated conserved quantum number T_a

$\hat{T}\Psi(r,t) = \Psi^*(r,-t) \Rightarrow$ not hermitian and not linear

$e^{\frac{i}{\hbar}(\vec{p}\cdot\vec{r}-Et)} \xrightarrow{\hat{T}} e^{-\frac{i}{\hbar}(\vec{p}\cdot\vec{r}+Et)}$

- combined with P invariance, transition from initial state i to final state f is identical to opposite transition
- $\boxed{a+b \to c+d \text{ same rate as } c+d \to a+b}$

$\begin{cases} \bullet\ \vec{p}, \vec{L}, \vec{B} \text{ flip sign} \Leftarrow \text{associated with movement} \\ \bullet\ \vec{x}, E, \vec{E} \text{ don't flip sign} \Leftarrow \text{static} \end{cases}$

CP – together conserved in weak interaction? not exactly but good approximation

CPT – conserved in weak interactions!
– only possible single particle eigenstates are those at rest : $CPT\,\Psi(\vec{r},t) = \bar{\Psi}(-\vec{r},-t)$

$\Rightarrow m_p = m_{\bar{p}}$: particle / antiparticle masses are equal

TM

Local symmetries

- what if transformation varies depending on position?
  $f(\vec{r}) \to f(\vec{r}) + h(\vec{r})$
- in SM: $U(1)_Q \times SU(2)_L \times SU(3)_C$

  color   electroweak

  = gauge symmetries

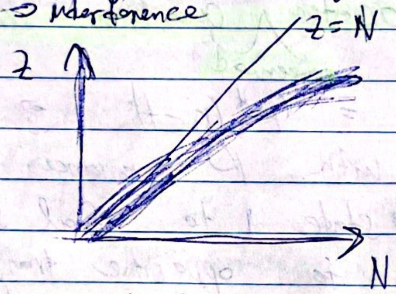
⇒ associated conserved current and charge
  e.g. electromagnetism → electric charge conservation

↳ more later

Nuclear Physics

- size: de Broglie wavelength $\lambda = \frac{h}{p}$
  → interference

$A = Z + N \left\{ \boxed{\begin{array}{l} Z = \#p^+ \\ N = \#n^0 \end{array}} \right.$



- H, He most abundant
- afterwards periodic peaking pattern (even more likely (more stable) than odd)
- spike for Fe
  ↳ highest binding energy
  - end of fusion (↑Z until Fe)
  - end of fission (↓Z until Fe)

Bohr atom

- mass rule: mass of atoms $\approx \underbrace{(N+Z)}_{\in \mathbb{N}} m_H$
  - for each atomic mass: integer $\times m_H$
    but isotopes break it ⇒ atomic mass = avg over isotopes
    e.g. $m_{Ne} = 20.2\, m_H$ ⇒ not integer

Neutron discovery → Chadwick

$Po \xrightarrow{\alpha} Be \xrightarrow{n} I_{\text{paraffin}} \to p^+$

50 MeV energy ⇒ too high for γ radiation

⇒ mass of atom:

$$\boxed{M_X = Zm_p + Nm_N + Zm_e - \frac{E_{bind}(A,Z)}{c^2}} \approx Am_p$$

$\{ \frac{E_{bin}}{c^2} \ll m_p,\ m_p \approx m_n,\ m_e \ll m_p \}$

Binding energy

- maximum per nucleon for ^{56}Fe in nature (but in reality can make weird stuff ^{56}Ni with slightly higher)
- A below: can do fusion in stars
- A above: can do fission, come from supernova

- strong force to overcome elmag. repulsion between protons to bind the nucleus together

Short range! ⇒ problems with stability for large nuclei

~1 fm

↳ Yukawa: $U(r) = -g^2 \frac{e^{-r/R}}{r}$

↳ binding of protons / neutrons

- gluons exchange betw...
- ≈ virtual pion exch...

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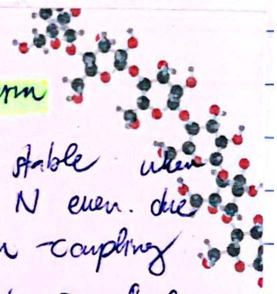


Pairing term

⇒ most stable when Z and N even. due to spin coupling

$\uparrow\downarrow\ \uparrow\downarrow > \uparrow\ \uparrow$

Semi Empirical Mass Formula

SEMF: Liquid drop model: nucleus ≈ drop of incompressible fluid

$$\boxed{E_{bind} = a_V A - a_S A^{2/3} - a_C Z^2 A^{-1/3} - \frac{a_a (Z-N)^2}{4A} + \frac{\delta}{\sqrt{A}}}$$

pairing term

$\boxed{V \propto A}$ $\boxed{S \propto A^{2/3}}$

Volume

$\boxed{R \propto A^{1/3}}$

attraction among nearest neighbours ⇒ only $\propto A$

but $(A-1)A$ if all (= # pairs)

Surface tension

attraction only from nucleus, missing neighbours on surface

↓ spherical shape to minimize

Coulomb term

↳ repulsion between protons. $R \propto A^{1/3}$

↳ $\propto \frac{1}{R}$

↳ $E_{Coulom} \propto \frac{Z(Z-1)}{R} \propto \frac{Z^2}{A^{1/3}}$

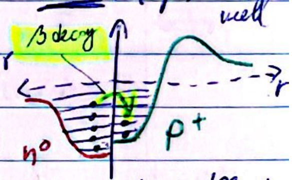
↳ large increase for large Z ⇒ N > Z

$U = \frac{3}{5} \frac{1}{4\pi\varepsilon_0} \frac{Q^2}{R} = \underbrace{\frac{3e^2}{20\pi\varepsilon_0}}_{a_c} \frac{Z^2}{A^{1/3}}$

replace Z^2 by $Z(Z-1)$ because doesn't make sense for Z=1

Assymetry term

↳ fermions in potential well



⇒ n^0 and p^+ different potential wells ⇒ off-set in energy levels ⇒ assymetry

- relevance of p-n assymetry decreases with A

$a_V = 15.67$ MeV

$a_S = 17.23$ MeV

$a_C = 0.714$ MeV

$a_a = 93.15$ MeV

$$\delta = \begin{cases} +11.2\,\text{MeV} & \text{, even } Z \text{, even } N \\ 0 & \text{, odd } A \text{ (Z even, N odd; Z odd, N even)} \\ -11\,\text{MeV} & \text{, odd } Z \text{, odd } N \end{cases}$$

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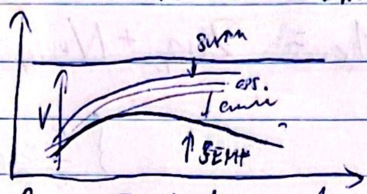
→ liquid drop model: ⇝ recovers binding energy curve



↳ breaks a bit for low Z but works well for higher Z

- to find stability curve (deviation from Z = N).

$\max\left[E_{bin}\left(\frac{N}{Z}\right)\right] \Rightarrow \boxed{\frac{N}{Z} \approx 1 + \frac{a_c}{2a_A} A^{2/3}}$

⇒ for larger A ⇒ N > Z for symmetric
⇒ for small A ⇒ N ≈ Z

$\max\left[\frac{E_{bin}}{A}\right] \Rightarrow$ A = 63 copper (SEMF prediction) vs. 62 nickel (reality)

↳ largest binding E per A
↳ vs. Fe: max E_{bin} per nucleon & avg over isotopes

Radioactive decay

In general:

- exponential decay $\frac{dN}{dt} = -\lambda N$

$$\boxed{N(t) = N_0 e^{-\lambda t}}$$

- mean lifetime $\boxed{\tau = \frac{1}{\lambda}}$
- half-life time $\boxed{t_{1/2} = \tau \ln(2) = \frac{\ln(2)}{\lambda}}$

- decay is possible when energy can be released
  - particle with higher mass
  - available decay method = interaction
  - lower mass particles to decay to

☆ $n \to p^+ + e^- + \bar{\nu}_e$ possible because $m_n > m_p + m_e + m_{\bar{\nu}_e}$

$\Delta E = 0.782$ MeV

- α decay : ${}^4_2He$
- fission - split of the atom
- β decay - e^- / e^+
- γ decay - photons

α, cluster decay, fission ↓ size of emitted object

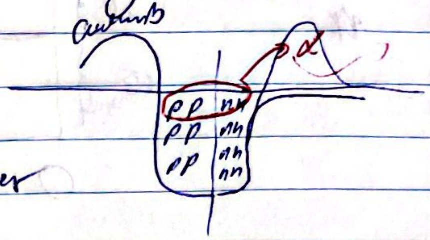
- electron capture

1

α-decay

- for large atoms, strong force can't hold together
- nucleus falls apart by emitting α particle
- Desintegration energy

$$E = (m_{init} - m_{final} - m_\alpha)c^2$$



- α: very strongly bound over other options

Ex $\quad {}^{232}U \longrightarrow {}^{228}Th + {}^{4}_{2}\alpha$

$232.0372 - 228.0287 - 4.0026 = 0.0059\ u > 0$ ✓ happens because it gains energy

${}^{232}U \longrightarrow {}^{231}Pa + p$

$232.0372 - 231.0359 - 1.0078 = -0.0065\ u < 0$ ✗

- α gains 4-9 MeV KE but there's still $\approx$ 25 MeV Coulomb wall : $V_{Coulomb} = Z(Z-2)\frac{\alpha}{r}$
  - $\Rightarrow$ not enough energy to leave :(
  - $\Rightarrow$ requires tunneling — takes time for probability to do the thing
- decay rate: $\lambda_\alpha = P(\alpha)\frac{v_0}{2R}e^{-G}$
  1. α particle has to form — probability $P(\alpha)$
  2. α has to reach the barrier : $t = \frac{v_0}{2R}$
  3. α has to cross the barrier : probability e^{-G} — Gamov factor

2

Fission — nucleus splits in 2

- Consider ellipsoidal shape of nucleus $\rightarrow$ how does this effect energy?
  - $\Rightarrow$ if energetically favourable $\Rightarrow$ ~~split~~ continue and split

$r_z = R(1+\varepsilon)$
$r_{x,y} = R(1-\varepsilon)$
$\Rightarrow$ surface: $E_S = a_S A^{2/3}(1 + \frac{2}{5}\varepsilon^2)$
coulomb: $E_C = a_C Z^2 A^{-1/3}(1 - \frac{1}{5}\varepsilon^2)$

$\Rightarrow$ total energy difference $\Delta E = \frac{\varepsilon^2}{5}(2a_S A^{2/3} - a_C Z^2 A^{-1/3})$

if $\Delta E > 0 \Rightarrow$ deformation disfavoured (costs energy)

if $\Delta E < 0 \Rightarrow$ deformation favoured (gains/releases energy)

$\Delta E = 0$: $\frac{Z^2}{A} = \frac{2a_S}{a_C} \approx 48 \Rightarrow$ e.g. $Z = 114$, $A = 270$ } spontaneous fission

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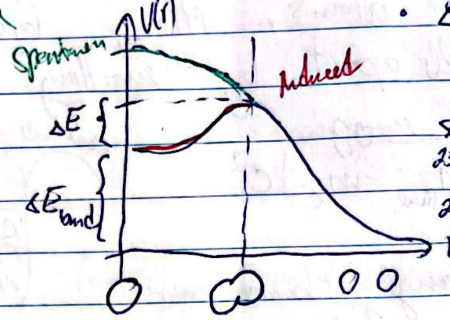
Spontaneous & induced fission

↳ rare

• $\Delta E < 0$



• $\Delta E > 0$: need to add energy to nucleus

$\Delta E = 5.5\,MeV$ for U

^{238}U: $E_{bind} = 4.6\,MeV$ ⇒ needs n with $E > 0.9\,MeV$

^{235}U: $E_{bind} = 6.4\,MeV$ ⇒ any neutron starts fission

3 β-decay

weak force

β^-: $n \to p^+ + e^- + \bar{\nu}_e$ — also free

β^+: $p^+ \to n + e^+ + \nu_e$ — only in proton

• α: strong vs elmag balance

vs. β – weak force

• allowed when energy is released

– leaves A const., changes Z by ±1

$$M_x = \alpha A + \beta Z + \gamma Z^2 + \delta A^{-1/2}$$ ⇒ quadratic for fixed A

β-decay to the center of parabola

4 Electron Capture ~ β^+ decay

but related:

$$p^+ + e^- \to n + \nu_e$$ (compete with β^+)

electron captured by proton

(from inner shell)

5 Neutron/proton capture/emission

6 γ (Gamma) decay

– usually biproduct in other processes

– from available energy levels in nucleus (excited states)

– usually after previous α, β decays

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Nuclear size & shape

· SEMF assumes size/shape of nucleus - but what are they?

Reaction types

Elastic ✶ $a + A \rightarrow a + A$ only modifies momentum

Inelastic ✶ $a + A \rightarrow a + A^*$ modifies at least one of the particles
✶ $a + A \rightarrow b + B$

- one particle gets excited
- both particles are changed

Direct reaction: initial → final state directly
✶ - short reaction time
✶ $p + {}^{16}O \rightarrow d + {}^{15}O$

Indirect / compound: intermediate state formed in the middle which quickly decays
✶ $a + A \rightarrow B^* \rightarrow c + C$
$\rightarrow B + \gamma$
$\rightarrow B + kn$

↳ usually multiple possible decay paths
↳ takes longer than direct
↳ smaller energy release than direct

$\frac{1}{\Delta E} \propto \Delta t$

$\delta E \delta t \approx \hbar$
10^{-34}
$6.2\,V\,10^{-19}$
10^{-19}
10^{-24}

more energy → shorter time
less energy → larger time

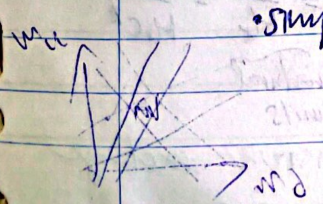
Nuclear symmetry: isospin

- 2 states of similar object ~ spin
  ↳ $p \sim n$ but different isospin

✶ nucleons $I = \frac{1}{2}$, $p^+ : I_3 = +\frac{1}{2}$
$n_0 : I_3 = -\frac{1}{2}$

· similar to angular momentum: deuteron $(p+n)$: $I = 0$ or $I = 1$ ($I_3 = 0$; $I_3 = -1, 0, 1$)

· simplifies interpretation of mass spectra $(m_p \approx m_n)$



Nuclear size and shape

$$\boxed{\sigma = \frac{dN}{dt}\frac{1}{\phi_{beam} N_{target}}}$$

cross-section

reactions per unit time

beam particles per unit time per unit area

scattering centers $= n_{target} \cdot V$

number density

volume

- theoretical basis for cross-section ⇒ Fermi's Golden rule
  - probability amplitude $\mathcal{M}$ (matrix element) for interaction to move from initial $|i\rangle$ to final $|f\rangle$ state

$$\mathcal{M}_{fi} = \langle f | H_{int} | i \rangle = \int \Psi_f^* H_{int} \Psi_i \, dV$$

final state, interaction Hamiltonian, initial state

  - transition probability $\Gamma_{i \to f} = 2\pi |\mathcal{M}_{fi}|^2 \rho(E_f)$

probability, density of states in final state $|f\rangle$

Scattering with Coulomb potential

- particle - nucleus scattering
  ↳ with Z protons

⇒ nucleus has potential determined by density of states:

$$\rho(x) = Ze\, f(x)$$

charge of nucleus, distribution of charge in the nucleus

↓

transition probability: $\langle \Psi_f | H_{int} | \Psi_i \rangle \propto Ze \underbrace{\int f(x) e^{iqx} d^3x}$

measure dependence of interaction on energy ⇒ test structure of object

form factor: $F(q^2)$ = Fourier transform of charge distribution

heavy particle (Yukawa potential)

$$V = -\frac{g^2}{4\pi}\frac{e^{-r/R}}{r}$$

coupling (here: weak interaction)

$$\Rightarrow F(q^2) = -\frac{g^2}{q^2 + M_x^2}, \quad \underbrace{R = \frac{1}{M_x}}_{\text{in natural units}} = \frac{\hbar c}{M_x c^2}$$

- range
- originally for strong force but holds for any force with massive exchange particle ⇒ makes the force have range $< \infty$

$m_{exchange} \approx 100\,MeV$
↓
pions

Nuclear charge distribution

• compare electron - nucleus scattering (Mott)
vs. alpha - nucleus scattering (Rutherford)

angles

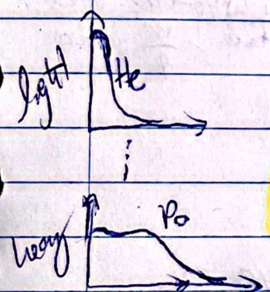
$$\left(\frac{d\sigma}{d\Omega}\right)_{exp} = \underbrace{\left(\frac{d\sigma}{d\Omega}\right)_{Mott}}_{\downarrow} |F(q^2)|^2 = \left(\frac{d\sigma}{d\Omega}\right)_{Ruther} \underbrace{\left(1-\beta^2 \sin^2\frac{\theta}{2}\right)}_{\text{spin correction}} |F(q^2)|^2$$

experiment

• Mott scattering equation works quite well for a certain range of energies but eventually breaks down (point charge approach breaks down)
⇒ if we use a lot of energy, we are probing the inside structure of the nucleus (or whatever we're probing)

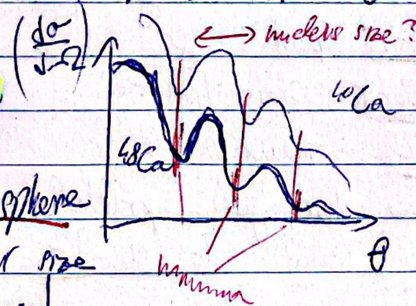
? correction to the equation above?
1. Determine form factor $|F(q^2)|^2$ based on $\left(\frac{d\sigma}{d\Omega}\right)_{exp}$
2. Take FT to determine nuclear charge distribution $f(x) = \frac{Ze}{(2\pi)^3} \int F(q^2)\, e^{-iqx}\, d^3q$

↳ useful for limited info
⇒ compare to example distribution and corresponding form factors
⇒ nuclei best described as a sphere with diffuse surfaces



NB: for scattering: best use fundamental particles otherwise our probe can also be probed and interact with its internal structure → fall apart

• oscillating structure + drop off
~ diffraction on hard sphere
→ minima from nuclear size (already for hard sphere)



↳ location of minimum similar
⇒ protons determine this and those are the same
↳ neutrons only form "skin"

• mean radius r^2
- lepton scattering → charge radius
- neutron scattering → matter radius

• for nuclei $r^2_{charge} \neq r^2_{matter}$

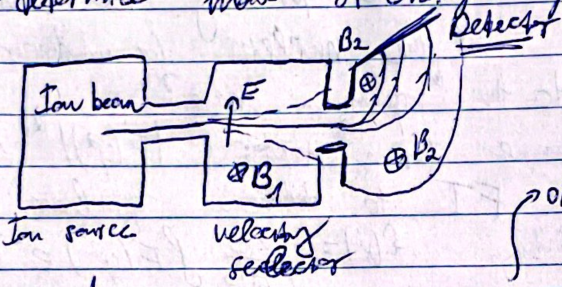
⇒ protons and neutrons are not homogeneously distributed

☆ ^{40}Ca vs. ^{48}Ca
• r^2_{charge} equal
• $r^2_{matter}(^{40}Ca) < r^2_{matter}(^{48}Ca)$

Experimental Methods

Spectrometers

• determine mass of charged particles using Lorentz force



↗ velocity selection not done!
of relativistic particle: $\frac{p}{q} = Br$

② Mass measurement:

$$\frac{mv^2}{r} = qvB_2 \Rightarrow \boxed{\frac{m}{q} = \frac{B_2 R}{v}}$$

based on radius of curvature we can find $\frac{m}{q}$ ratio

① velocity selector: $F_{net} = F_E - F_B = qE - qvB_1$

if $F_{net} = 0 \Rightarrow v = \frac{E}{B_1}$

↳ to make sure we have particles of known velocity to use the formula above

• to determine decay products of excited states
- atomic spect. : γ
- nuclear : photons/particles
- particle : new particles

⇒ use decayed particles to determine excited state and its structure

Setting up a subatomic experiment

Source → Interaction → Detector

↳ accelerator ↳ target

natural { ↳ cosmic rays, ↳ nuclear decay }

↳ uncontrollable

↳ limited options for high energy

ways of colliding particles < Fixed-target : beam on stationary target
Collider : beam on beam

Collider: $E_{CM}^2 = \left(\begin{matrix} E_{beam} + E_{beam2} \\ \underbrace{\vec{0}}_{\text{if same but opposite momenta of the beams}} \end{matrix}\right)^2 = 4E_{beam}^2 \Rightarrow \underline{E_{CM} = 2E_{beam}}$

vs

Fixed-target: $E_{CM}^2 = \left(\begin{matrix} E_{beam} + m_{target} \\ \text{non-zero: } \vec{p}_{beam} \end{matrix}\right)^2 = (E_{beam} + m_{target})^2 - p_t^2$
$= m_b^2 + m_t^2 + 2E_b m_t$

$\Rightarrow$ useful energy $= \sqrt{2E_b m_t} \ll$ collider

especially if KE >> mass energy

Accelerators

- charged particles → accelerate them with E&M fields
- provide controlled source of particles
  - choose energy
  - choose particle type
  - ⇒ able to test specific interaction

Linear accelerators

- focused beam
- apply voltage ~~at~~ to set of drift tubes ⇒ accelerate to first tube
- while in first tube, flip voltage ⇒ accelerate to second tube

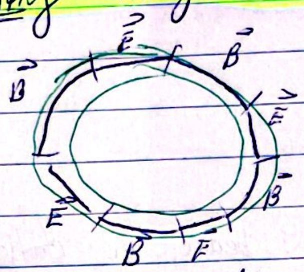
Cyclotron

- not restricted by length of a tube
-

switching direction of $\vec{E}$ between Dee's ⇒ acceleration ⇒ increase in radius

Synchrotron

- vs. cyclotron: keep radius const. and vary $\vec{B}$
- alternate accelerating (electric field) and
  bending (magnetic field)
  but adjusting magnetic field to keep radius stable



- also using magnets (magnetic optics) to focus the beam
- limited by synchrotron radiation ($P \propto \frac{1}{m^4}$ for relativistic)
  ⇒ reasons to use linac's for electrons (light ⇒
  ⇒ synchrotron rad. worse for electrons)
- slower particles get larger kick, faster particles less so ⇒ bunch particles together
- minimise velocities transverse to the beam
  (to keep effective area A narrow)
- luminosity (beam intensity)

$$\boxed{\mathcal{L} = n_{bunches}\frac{N_1 N_2 f_{rev}}{A}}$$

$n_{bunches}$: # ~~particles~~ bunches

N_1, N_2: # particles per bunch

f_{rev}: revolution frequency (how often beams meet)

A: effective area of collisions (smaller = better)

Fixed target vs. Collider

Fixed target:
- simpler operation
- high density target
- many possible targets
- high luminosity

Collider:
- needs precise manipulation of beams
- low density ⇒ low luminosity
- high energy

☆ LHC: $m_{CoM} = 13.6\,TeV$ ($2 \times 6.8\,TeV$)
$\mathcal{L} = 10^{34}\,cm^{-2}s^{-1}$ ($f \approx 40\,MHz$)

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Interaction with matter – microscope process

- electrically charged particles ⇒ macroscope detection?
  ⇝ direct/optical signal
- electrically neutral ⇒ first induce reaction to produce charged particles

Cloud chamber

- condensation induced by ionised atoms in gas
- ~~do~~ to detect, convert microscopic to macroscopic process (here a cascade)

Interactions	Detection
Photoelectric effect, EM:	Scintillators, photomultiplier
Weak interaction (neutrinos, DM)	• needs large masses ⇒ water, ice, ...
Strong interaction	

Ionisation – elmag. : hitting electrons out of atoms

- main interaction for charged particles
- easy to see, particles don't lose a lot of energy and isn't destroyed

scattering / ⇒ statistical process

- Bethe-Bloch equation

const. · electron density in medium · relativity · correction factor (small)

$$-\frac{dE}{dx} = \frac{D\, q^2 n_e}{\beta^2}\left[\ln\left(\frac{2m_e c^2\beta^2\gamma^2}{I}\right) - \beta^2 - \frac{\delta(\gamma)}{2}\right]$$

energy loss when going through a medium

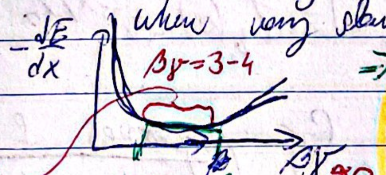
charge of particle

ionisation potential $\approx 10 Z$ eV for $Z > 20$

$\beta := \frac{v}{c} \leq 1$

⇒ quadratic drop off at low velocities } ⇒ Bragg peak

⇒ most energy lost when very slow already



! ⇒ for low speed, large drop off, for very fast, ~~[illegible]~~ middle drop off, minimum inbetween

small energy loss → this is where we want to do this

~ roughly const. loss of energy at minimum = minimum ionisation region

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Radiation (bremsstrahlung) – much less often than ionisation but larger effect

· electron – nucleus interaction

$$-\frac{dE}{dx} = \frac{E}{L_R} = \frac{4\hbar^2 E}{m^2c^2} Z(Z+1)\,\alpha^3\, n_a \ln\left(\frac{183}{Z^{1/3}}\right)$$

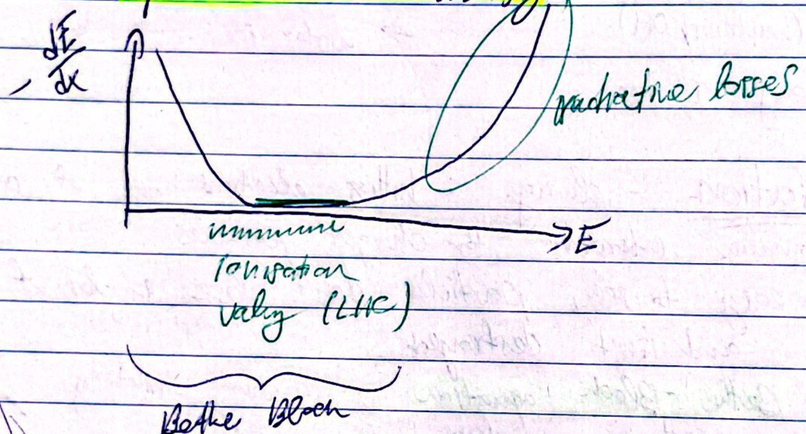
energy loss

radiation length = typical distance which particle travels until it interacts

atom density in medium

strong dependence on $\frac{1}{m^2}$ and Z^2+Z ⇒ dominant at many energies for electrons, subdominant for other particles (~~but can be~~ relevant for μ in lead only for $E \geq 140\,GeV$)

⇒ quick loss of energy



Interaction length

· define average length before significantly interacting
· beam intensity will change by

$$\boxed{\frac{dI}{dx} = -\frac{I}{X_0}}$$

$X_0 = L_R$ = radiative loss

$$L = \boxed{X_0 = \frac{1}{n\sigma_I}}$$

$$\Rightarrow \boxed{I_{beam}(x) = I_b(0)\, e^{-x/X_0}}$$

sometimes interaction is destructive, sometimes particle leaves (just lose energy)

↳ essential for properties of detector
↳ very relevant for L_R (radiation) ~~length~~ and strong force

photons get destroyed this way

Photons $L_\gamma = \frac{9}{7} L_R$

· lose energy through interactions in same way as electrons ⇒ especially happens when hitting nuclei.
· for ↑E (> units MeV) ⇒ pair production dominant

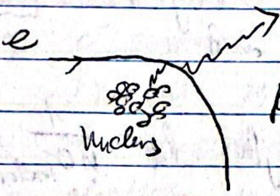
Charged particles – direct detection
Neutral particles – must be detected indirectly $\Rightarrow$ decay products

Any undetected energy lost to interactions can worsen the measurement

Elmag interactions

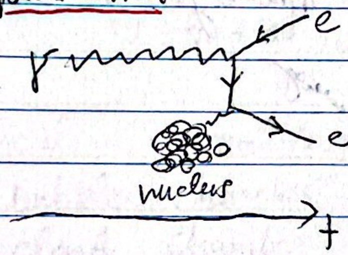
Ionisation – light interaction, easy to eject electrons, occurs a lot, constant rate while going through detector

Bremsstrahlung:



path of particle bent by nucleus $\Rightarrow$ loss of lots of energy

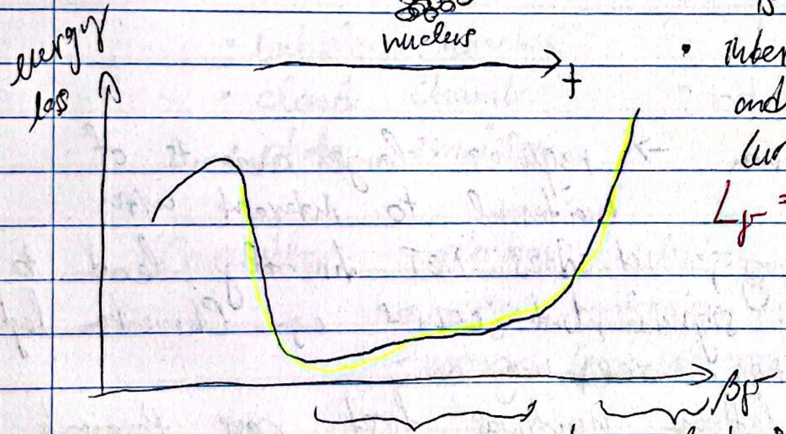
Pair production:



- $\gamma \to ee$ in vicinity of nucleus
- main interaction for photons is pair production
- interaction length of photons and electrons is related (unseparable): $L_\gamma = \frac{9}{7} L_R$

$L_R = X_0$ = radiative loss

$L_R = \frac{1}{n\sigma}$



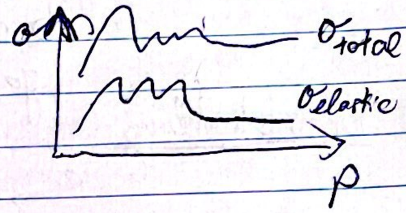
most particles: minimum ionization region – small energy loss

electrons suffer many radiative losses (Bremsstrahlung)

Strong interactions

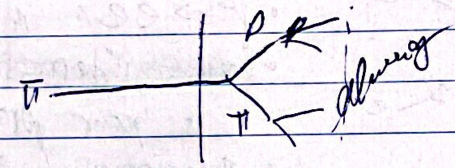
☆ elastic $\pi^- + p^+ \to \pi^- + p^+$ } always both
inelastic $\pi^- + p^+ \to K^0 + \Lambda^0$ } happen

↳ for higher energies, more different processes
↳
• resonances, inelastic cross-section dominates for E > 3 GeV



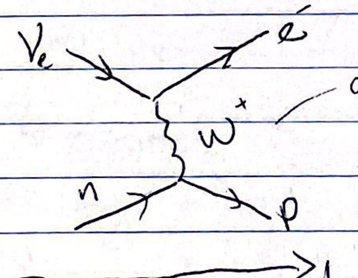
• generic description based on interaction lengths
$\boxed{l_c = \frac{1}{n\sigma_{total}}}$ and
absorption length $\boxed{l_a = \frac{1}{n\sigma_{inelastic}}}$

⇒ does NOT lead directly to ell/ung signal



Weak interactions

• less common → requires large amounts of material to interact with
• as for strong, it does NOT directly lead to electronic signal, but causes e.g. Cherenkov light
☆ sea, Antarktica ice, mountain
⇒ letting neutrinos first pass through lots of water / ice / ... underneath e.g. a burried detector to detect end products



or Z but that only results in scattering ⇒ provides less information

Detectors

- scintillation : $e \to \gamma$ $\Rightarrow$ detect γ
- $\vec{E}$ field : move e^- to electrode

+ signal enhancement ↪ can also be used for acceleration and then creating an avalanche (easier to detect than a single particle)

- microscopic → macroscopic
  - easily excitable material (close to threshold)
    e.g. gas that's easily ionised
  - amplification process
    e.g. e^- current s.t. primary e^- is accelerated and ionises secondary e^- (avalanche)
  - detection of amplified process
    e.g electrode records avalanche of secondary e^-
  - read out data
    e.g. front-end, data

Historical - visual /manual reconstruction
- bubble chamber
- cloud chamber
- photo emulsion

Currently - electronic detectors (precision, speed)
- mostly ionisation, some scintillation, Cherenkov radiation
- many technologies available

(?) What detector?
- consider process and particle measured (elmag /strong /weak)
- property to measure : energy (destructive)
  + resolution momentum nondestructive (only for charged)
- source - interaction rate /detection rate / data rate
- cost (money and material)
- radiation hardness

layers
↳ multiple detectors

scintillators - time
gas detectors - position / momentum (if $\vec{B}$)
semiconductors - position / energy loss / momentum (if $\vec{B}$)
Cherenkov - velocity

} similar but gas cheaper, older, less precise

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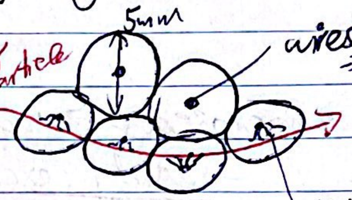
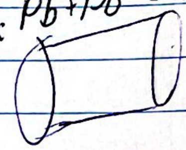
Gas detectors - tracking

- charged particles passing through gas causes ionisation
  - collect ionisation products (ions, electrons) on electrodes
  - historically visualise trajectory
- detect pulse at anode from drifted ions/electrons
- with sufficient voltage, avalanche in gas → → amplification



- use timing information to provide information on track hit within gas → up to resolution 100 μm

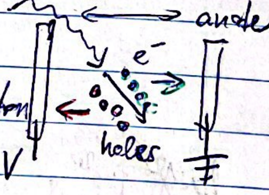
Time projection chamber - used in ALICE : $Pb^{82+} + Pb^{82+}$ collisions



- full volume of detector
- ionisation detectors drift to side
- <u>slow</u> process, many "hits" per track ⇒ <u>very</u> precise
- measure deflection in magnetic field for momentum, change in curvature for E-loss ↳ from energy loss vs time → place on βγ plot

Semiconductors - tracking

- semiconductor from which particle can easily free electrons (1-4eV) → less E than ionisation
- forms electron-hole pair, analogous to electron-ion pair in gas
- $\vec{E}$ field separates charges → collected at electrodes
- #pairs ∝ E-loss, particle charge
- very precise, little material, high resolution

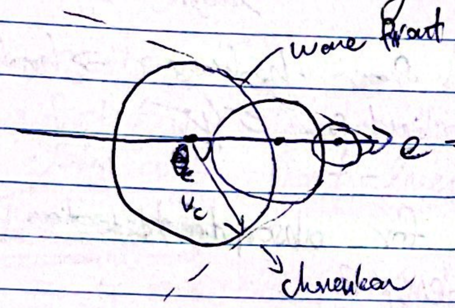


- strip (1D), radiation hard, cheaper)
- pixel (2D, more sensitive, costly)
- often used at centre of detector at LHC ⇝ small area, many particles

Cherenkov radiation

- if particles goes through medium with refraction index n with velocity $\underline{v}$ > velocity of light in medium $v_m = \frac{c}{n}$ (still always less than c) ⇒ emission of light

☆ $n = 1.33$ water
$n = 1.0002 - 1.001$ · gas — good enough for fast particles
$n = 1.5 - 1.75$ glass — useful for slow particles



- angle of light emission depends on velocity
  $\cos\theta_c = \frac{1}{\beta n}$

⤳ Ring-imaging Cherenkov detectors
- detect ring produced by particles with angle θ_c to determine velocity
- combine with momentum/energy measurement, obtain mass/particle ID
  ↳ necessary in LHC (alone doesn't do much) but self-standing for antarctica measurements of neutrinos

Scintillators

- uses excitation of atoms (not ionisation)
  → excited atoms decay → produce scintillation light
- detect with photomultiplier tubes (PMT)
- de-excitation can be very rapid ⇒
  ⇒ useful as triggers, time-of-flight (velocity → particle ID)
- efficient at detecting low-energy particles
- flexible & cheap, but not as precise

Calorimeters

- make particles interact with material → showers
- fully absorb particle for detection, measure energy (especially useful for neutral particles)
- optimised for e^-/γ or hadrons ↓

short radiation interaction length e.g. Pb | short hadronic interaction length e.g. Fe | using secondary particles

- electronic calorimeter

separates e^-/γ from hadrons → hadrons pass through while e^-/γ

homogeneous
- one material for absorption + detection
- e.g. scintillating crystal

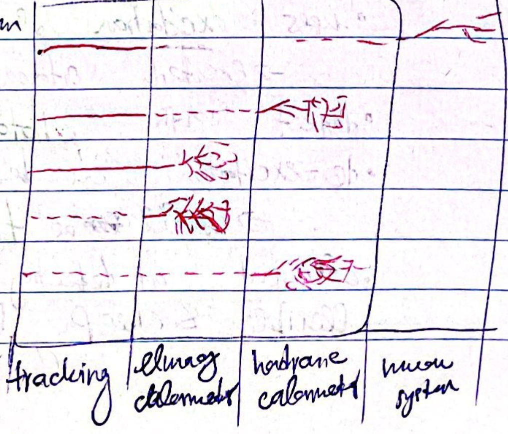
Shashlik / sandwich
- interleave absorber + detector – absorber – detector – ...

absorber ↓ lead Pb; detector ↓ scintillator

- can lead to loss of particles / energy in absorber
- if detector is scintillator, it detects light from charged particles in showers $\sim E_{initial}$
- resolution improves with energy (opposite to tracking)

Layered Detectors – detect as many particles as possible

- muons – tracking + muon system ↳ 99%
- charged hadrons
- electrons
- photons
- neutral hadrons



Strong Interaction

- e^-, p^+, n^0, γ
- accidentally from cosmic rays : $\mu, \pi, (\nu)$
- antimatter : e^+

⇝ things changed when strange particles were observed
— from cloud chamber cosmic ray observations

V-like: $K^0 \to \pi^+ + \pi^-$
K-kink: $K^+ \to \mu^+ + \nu_\mu$
↳ lifetime of order ns

- 1950's : accelerator tech : 1GeV ⇒ many new particles
- first long-lived hadrons discovered
- later also short-lived (intermediate, not observed directly → peaks in histograms)

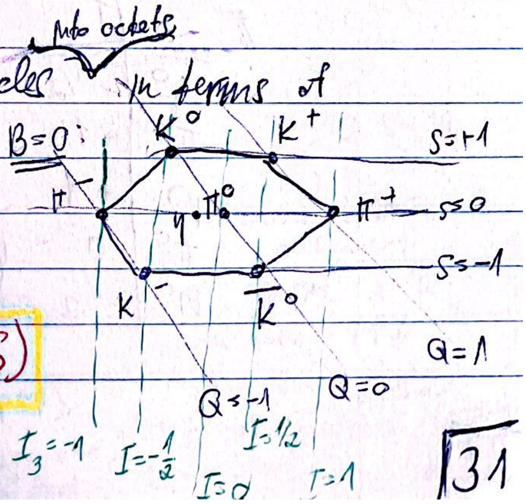
Isospin

- in nuclei, protons and neutrons behave similarly ⇒ imagine as same things with ~~them~~ associate quantum numbers : isospin
- nucleons : isospin $\frac{1}{2}$ → projections : $I_3 = +1/2$ p^+, $I_3 = -1/2$ n^0
- similar properties hold for other observed particles
  ☆ π : $I = 1$ and $I_3 = +1$ for π^+, $I_3 = 0$ for π^0, $I_3 = -1$ for π^-
- conserved quantity in strong interaction (but not in elmag / weak)
- ~~mesons have integer isospin, baryons have half-integer isospin~~
- particles with similar isospin (in same multiplet) have similar masses

Observed initially but doesn't actually hold for Σ, Ξ, π^0

⇝ we can organise particles into octets in terms of
- electric charge Q
- strangeness S
- baryon number B
- isospin I_3

↳ related :
$$\boxed{Q = I_3 + \frac{1}{2}(B+S)}$$
$$\boxed{I_3 = \frac{1}{2}(N_u - N_d)}$$



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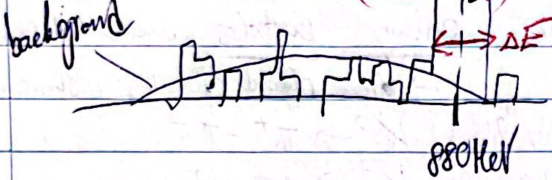
Short-lived hadrons

- existence of intermediate particles can be inferred from

★ $K^- + p \rightarrow X^- + p$

$c^4 m_X^2 = (E_K + E_\pi)^2 - (p_K + p_\pi)^2$



$\approx 50\,MeV \rightsquigarrow$ very short-lived

$\rightsquigarrow$ long-lived order of keV

$\underbrace{\Delta E}_{\text{width of the particle}} \Delta t \approx \frac{\hbar}{2} \Rightarrow \Delta t \approx \frac{\hbar}{2\Delta E}$

$\tau \approx 10^{-23}\,s$

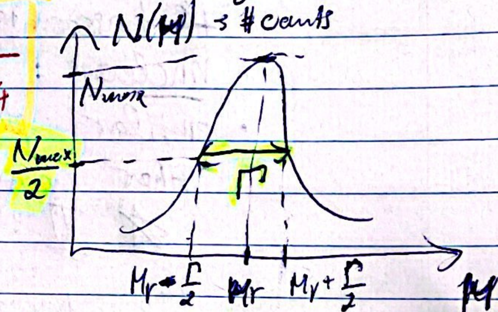
$\Gamma \approx 100\,MeV$

$$\boxed{\Delta E \approx \Gamma \equiv \frac{\hbar}{\tau}}$$

- short-lived $\Rightarrow$ strong force $\Rightarrow X^- = K^{*-}$

- Shape of peak given by Breit-Wigner

$$\boxed{N(M) = \frac{C}{(M-M_X)^2 + \Gamma^2/4}}$$



	I_3
Δ^{++}	$3/2$
Δ^+	$1/2$
Δ^0	$-1/2$
Δ^-	$-3/2$

★ $\pi^+ + p \rightarrow \Delta^{++} \rightarrow \pi^+ + p$ (p: $I_3 = 3/2$)

$\pi^- + p \rightarrow \Delta^0 \rightarrow \pi^- + p$ ($I_3 = -1/2$; $-1/2$, $+1$)

excited particles

Δ^0 can also decay into $\pi^0 + n^0$

isospin favored $\Rightarrow$ more ~~proble~~ probable than $\pi^- + p^+$

	I_3
Δ^0	$-1/2$
π^-	$-1/2$
p	$+1$
π^0	0
n^0	$-1/2$

π^-, p: $\} +1/2 \neq I_3$ of Δ^0 ☹ $\rightsquigarrow$ weak/elmag

π^0, n^0: $\} -1/2 = I_3$ of Δ^0 ☺ $\rightsquigarrow$ strong $\Downarrow$ fast ✓

Quarks

- proposed by Gell-Mann and Zweig
- ~~p~~ fractional charges
- isospins, ~~ch~~ strangeness, ... quantum numbers

	Q	I_3	S
u	$+2/3$	$+1/2$	0
d	$-1/3$	$-1/2$	0
s	$-1/3$	0	-1

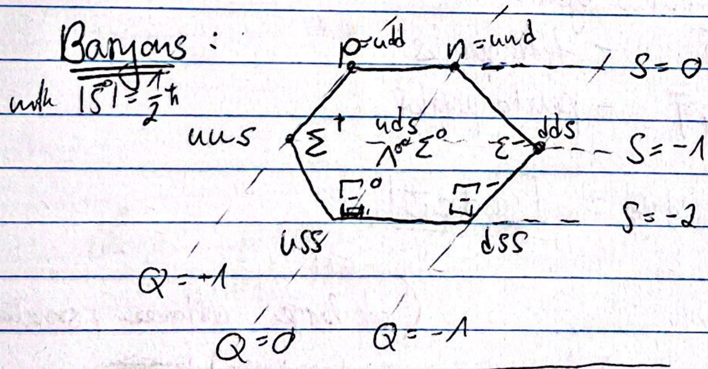
- if isospin and/or strangeness violated by weak force but conserved in strong force
- also charm C, beauty B, top T
  ⇝ additional quantum numbers
- $m_d > m_u$
- top quark is too heavy to hadronise
  → decays before forming hadrons

π^0 : $S = 0$: $\frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$

η : $S \sim 1/3$: $\frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$

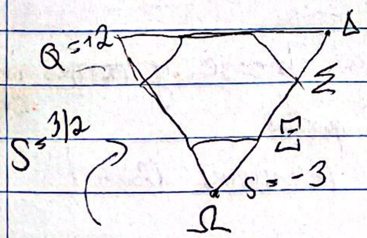
η' : $S > 1/3$

Baryons:
with $|S| = \frac{1}{2}\hbar$



uds
Λ^0, Σ^0
$u\uparrow d\uparrow s\downarrow$ vs. $u\uparrow d\downarrow s\uparrow$ } leads to different masses

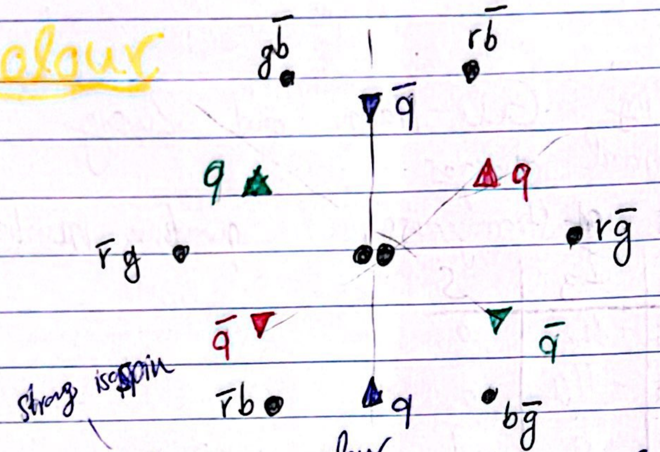
$\boxed{P_b = 1 \text{ for } L=0 \text{ baryons}}$ ⇒ ~~sss~~ for $L=0$, $S=1/2$
↓
symmetric under exchange
but to get $S = 1/2$ we need $\uparrow\uparrow\downarrow$
but then switching is not symmetric.



⇐ possible only for $S = 3/2$ ⇐
or $L > 0$ states

$|\uparrow\uparrow\uparrow\rangle$ but now $\Omega^- = |sss\rangle$ with $|\uparrow\uparrow\uparrow\rangle$
and $L=0$ ⇝ symmetric!!
→ Pauli → color

Colour



quarks

	I_3^c	Y^c
r	+1/2	+1/3
g	-1/2	+1/3
b	0	-2/3
	0	0

— colour hyperchange

antiquarks

	I_3^c	Y^c
$\bar{r}$	-1/2	-1/3
$\bar{g}$	1/2	-1/3
$\bar{b}$	0	2/3

⇝ stable state has 0 colour

⇝ no free quarks and only integer charges

- heaviest quark dominates a hadron
  - makes up ~~the~~ most mass
  - decays fastest

Exotic states

- new states ; besides $q\bar{q}$, qqq, $\bar{q}\bar{q}\bar{q}$; discovered in last few years :

$\boxed{\begin{array}{ll} q\bar{q}q\bar{q} & \text{- tetraquarks} \\ qqqq\bar{q} & \text{- pentaquarks} \end{array}}$

☆ $J/\psi - p^+$ state : $\boxed{uudc\bar{c}}$

QED x QCD

QED	QCD
• γ exchange	• gluon exchange
• massless, spin 1 boson	• massless, spin 1 boson
⇒ infinite range	but limited range because g carries own charge
• interaction with everything but ν	• interaction with quarks
• flavour universal (conserves flavour)	• flavour universal (conserves flavour)

(EM at large distance, smaller coupling) ~~between~~

• because gluons have the charge that they work on (colour), → additional Feynman diagrams with gluon self-interaction increase the strength

⇒ these additional diagrams enhance coupling instead of decreasing it ⇒ anti-screening

⟹ force is **stronger** at larger distances (low energies)

↳ works opposite than elmag which gets weaker at distance ⇒ screening

coupling weaker at high energies (short ~~..~~ lengths) $\alpha_{strong}(2\,GeV) \cong 0.32$, $\alpha_{strong}(100\,GeV) \cong 0.12$

problems for ~~low-level~~ low-energy processes:

⇝ higher order diagrams add more than lower order ⇒ we can't do ~Taylor series expansion in α like for the case of QED ($\alpha_s > 1$)

⇝ for ~~very~~ high energies we can, e.g. $\alpha_s(100\,GeV) \cong 0.12$ ⇝ secondary terms surpressed by order 10 etc.

⇒ good approximating series ☺

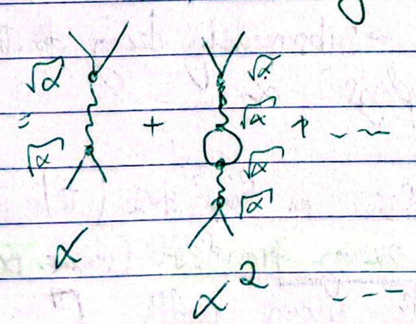
⇒ quarks "free" while ~~...~~ at high E (= short scale) e.g. within hadron, if moving ~~..~~ too far, pulled back (like spring) or break apart ⇒ jet production

• quarks break free flying off, (too much E)

⇒ form new hadrons

⇒ if many, jets of hadrons = high density bursts

α = probability of interaction (vertex) occuring ⇒ add up all α's from the whole process



if α < 1: higher order diagrams ~~..~~ approximately not necessary (insignificant)

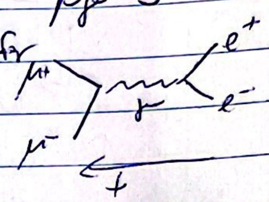
Counting quarks ~~&~~ with leptons

- cross-section drops off rapidly vs. $E_{CM} = \sqrt{s}$ ↑ momentum transfer $p_T = s$

Start with: $e^+e^- \to \mu^+\mu^-$ = well-known clean decay used for normalisation

$$\frac{d\sigma}{d\Omega} = \frac{\alpha}{4s}(1+\cos^2\theta) \quad , \quad \sigma = \frac{4\pi\alpha^2}{3s}$$

for



Now add quarks: $\sigma(e^+e^- \to q\bar{q}) = N_c Q_q^2\, \sigma(e^+e^- \to \mu^+\mu^-)$

↳ relative to $e^+e^- \to \mu^+\mu^-$

but hadrons form resonances → complicated

but we can: 1st test:

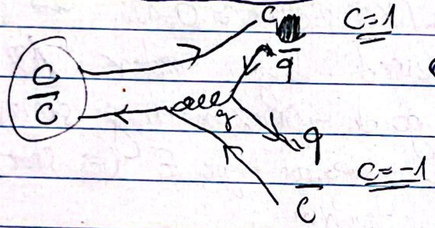
- angular dependence of cross-section which should be same for quarks and muons

⇝ worked fine for quarks ⇒ point particles ✓

$\boxed{\text{slide 29}}$ of Lecture 9

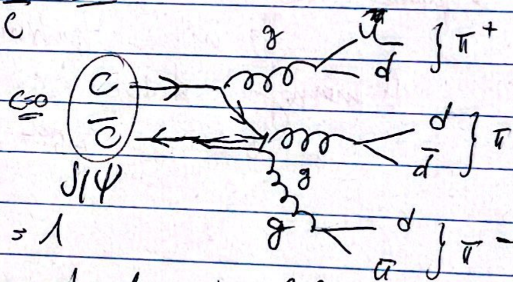
Lecture 10

☆ $C=0$



only possible if $c\bar{c}$ meson has sufficiently high energy.

If lower energy: $C=0$



lowest E meson

↳ $P_{J/\psi} = -1$ but $P_g^2 = 1$

$M_{J/\psi} < 2m_{c\bar{u}}$

⇒ J/ψ cannot decay to 2 gluons

⇒ also cannot decay to 1 gluon because 1 vertex cannot conserve both momentum & energy

⇒ $J/\psi \to 3$ gluons ⇒ subsequently decay to π's

⇒ 6 strong couplings α_s

↳ suppressed

↳ light particles in final state (π)

⇒ large energy transfer (so ~~same~~ α_s small)

⇒ small decay width Γ

⇒ large Δt

small energy emission favoured by strong force (α_s is large then)
⇒ if α_s is small for a decay, results in long life (also slow production rate)

34 (conserve

Deep inelastic scattering

- use e^- to probe p^+ (because quarks are charged)
- cross-section varies with energy, slow fall off at high energies
- size of structure depends on momentum

$\Rightarrow$ proton has more than one

- higher energy transfer $\rightarrow$ smaller scale probed $\rightarrow$ more virtual quarks

Conclusion

- gluons have colour charge $\rightarrow$ self interaction
- gluon coupling decreases with energy
- quarks only assymptotically free (at $\uparrow E$ or $\downarrow$ distance)
  $\Rightarrow$ confined to hadrons

Leptons & SM

$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$ — $m_\mu = 105.7\,eV$, $\tau_\mu = 2.2\,\mu s$

$\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau$
$\mu^- \bar{\nu}_\mu \nu_\tau$
$\pi^- \nu_\tau$
$\pi^- \pi^+ \pi^- \nu_\tau$

- to detect ν: need ~~beam~~ high flux or high E
  $\Downarrow$ increase σ
  $\Downarrow$ because weak force becomes stronger at higher energies

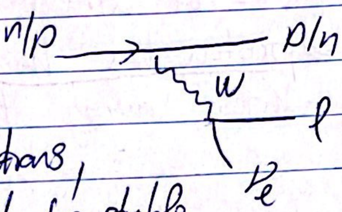
Weak interaction.

- weak (at low energies = low energy transfers)

β decay

- mediated by weak force

$\beta^-: n \to p^+ e^- \bar{\nu}_e$
$\beta^+: p^+ \to n e^+ \nu_e$

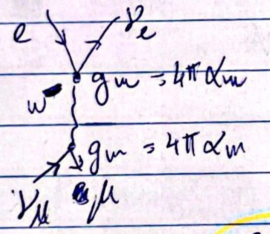


- without weak interactions, e.g. neutrons would be stable (they couldn't decay because strong/em doesn't permit)

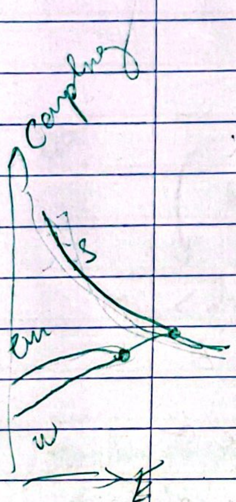
decay allowed, when energy is released

Z^0 and γ have same quantum numbers (interchangable up to mass and involvement of ν)

α_w = coupling of weak force to the particles



$A(\nu_\mu e \to \nu_e \mu) \propto g_w^2$
amplitude
$\Rightarrow$ observables : $A^2 \propto g_w^4$



$$\boxed{g^2 \to \frac{g^2}{q^2 - M_x^2}}$$

M_x: mass of exchange boson

$\boxed{q = \text{energy transfer in interaction}}$

for low energy transfer $q^2 \ll M_x^2 \Rightarrow \frac{g^2}{q^2 - M_x^2} \approx \frac{g^2}{M_x^2}$

$\Rightarrow M_x$ large for weak ($W = 80\,GeV$, $Z = 90\,GeV$)

$\Rightarrow$ weak interaction suppressed (less so in high E transfers)

↳ coupling isn't so weak, $\alpha_w \sim 0.58\,\alpha_{em}$
the mass fucks it over

$\boxed{\text{coupling is dimensionless}}$

★ Muon decay

effective coupling $\frac{g^2}{M_W^2}$

$[G_F^2] = E^{-4}$

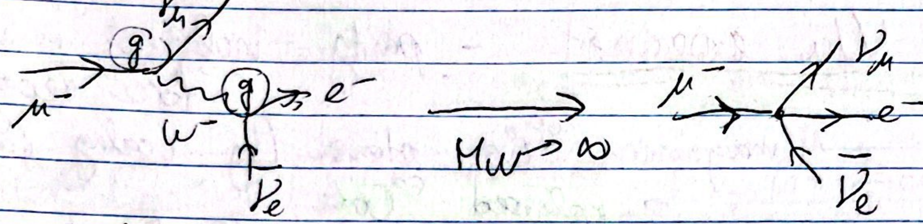
from dim. analysis $[\Gamma] = E$

$\Rightarrow \boxed{\Gamma = K\, G_F^2\, m_\mu^5}$

K: dimensionless constant depending on form of interaction

neglecting electron mass $\Delta E = m_\mu - m_e \approx m_\mu$

$\Gamma \propto m_{particle}^5 \Rightarrow$ larger for higher energy transfers (weak force get stronger at larger energies)



↳ extends to τ

★ Estimate of ratio of lifetimes:

$$\frac{\tau_\tau}{\tau_\mu} = \frac{BF(\tau^- \to e^- \bar\nu_e \nu_\tau)}{BF(\mu^- \to e^- \bar\nu_e \nu_\mu)} \frac{\Gamma(\mu^- \to e^- \bar\nu_e \nu_\mu)}{\Gamma(\tau^- \to e^- \bar\nu_e \nu_\tau)} = \frac{BF(\tau)\, m_\mu^5}{BF(\mu)\, m_\tau^5}$$

$$\boxed{\tau = \frac{BF}{\Gamma}}$$

Branching fraction
↳ probability fraction that it will decay via the path

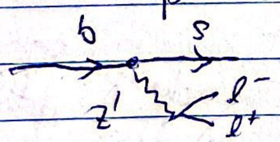
$$BF(\tau^- \to e^- \bar\nu_e \nu_\tau) = \frac{\Gamma(\tau^- \to e^- \bar\nu_e \nu_\tau)}{\Gamma_{tot}}$$

$$\boxed{BF(\text{particular decay}) = \tau_{particle}\, \Gamma(\text{particular decay})}$$

$BF_\mu = 1$

$BF_\tau \approx 17\%$ $\Rightarrow$ $\frac{\tau_\tau}{\tau_\mu} = 1.3 \times 10^{-7}$ prediction agrees with experiment!

★ Z' = like Z but able to change flavour of particles. ~~while~~ Z only works like ~photon



★ lepton universality

– all leptons are ~~universal~~ equivalent, couple same strongly to forces

CP

- parity P – invert space (also momentum $\vec{p}$, el. field $\vec{E}$, not spin)
- charge conjugation C – inverts "charge"
  - ↳ inverts all quantum numbers
  - ↳ but particles in same state (e.g. spin state)

Wu experiment – parity violation in weak force decay

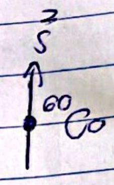
1) magnetize ^{60}Co atoms (by cooling down)
   ⇒ polarized ^{60}Co
2) observe decay $^{60}Co \rightarrow {}^{60}Ni^{*}\, e^- \bar{\nu}_e$

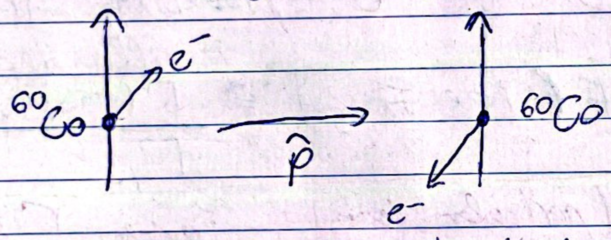
$$^{60}Ni^{*} \rightarrow {}^{60}Ni\, \gamma\gamma$$

⇒ γ from Ni^* reveal orientation of $\vec{S}$ in ^{60}Co

? favoured direction of e^- emission.
If parity is conserved, all angles equally likely ⇒ strong anisotropy

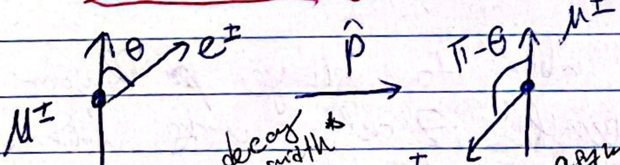
! spin unaffected by $\hat{P}$





↳ electrons opposite to direction of photons (related to direction of spin of ^{60}Co)

Muon decay



asymmetry parameter: $\xi_- = -\xi_+ = 1.00 \pm 0.04$ ⇒ maximal parity violation

*general probability of decay occurring

$$\Gamma_\pm = \int_{-1}^{+1} \Gamma_{\mu^\pm}(\cos\theta)\, d\cos\theta$$

decay width*

$$\boxed{\Gamma_{\mu^\pm} = \frac{1}{2} \Gamma_\pm \left(1 + \frac{\xi_\pm}{3} \cos\theta\right)}$$

angle dependence

~~Applying~~ $\hat{\mathcal{E}}$ operation

$\hat{P}$ |

$$P\Gamma_{\mu^\pm} = \frac{1}{2} \Gamma_\pm \left(1 + \frac{\xi_\pm}{3} \cos(\pi - \theta)\right) = \boxed{\frac{1}{2} \Gamma_\pm \left(1 - \frac{\xi_\pm}{3} \cos(\theta)\right)}$$

change sign ⇒ different decay rates!

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applying $\hat{C}$ operator

$$\boxed{C\,\Gamma_{\mu^\pm} = \frac{1}{2}\,\Gamma_{\mp}\left(1+\frac{\xi_{\mp}}{3}\cos\theta\right)}$$

$\Rightarrow \Gamma_{\mu^\mp}$

If C conserved, μ^+ and μ^- decay in same way

$\xi_- = \xi_+$ ✗ and $\Gamma_- = \Gamma_+$ ✓

but $\xi_- = -\xi_+$ $\Rightarrow$

$\Rightarrow$ charge conjugation violated

but

$$CP\,\Gamma_{\mu^\pm} = \frac{1}{2}\,\Gamma_{\mp}\left(1-\frac{\xi_{\mp}}{3}\cos\theta\right) = \frac{1}{2}\,\Gamma_{\mp}\left(1+\frac{\xi_{\pm}}{3}\cos\theta\right)$$

$$= \frac{1}{2}\,\Gamma_{\pm}\left(1+\frac{\xi_{\pm}}{3}\cos\theta\right) \quad \xi_- = -\xi_+$$

$= \Gamma_{\mu^\pm}$ $\Rightarrow$ CP conserved

If P and C conserved, $\xi_\pm = 0$

celulóza

Parity violation

If $\Gamma_{\mu^\pm} = \hat{P}\,\Gamma_{\mu^\pm}$ then parity conserved.

But we observe $\xi \neq 0$

$\Rightarrow \Gamma_{\mu^\pm} \neq \hat{P}\,\Gamma_{\mu^\pm}$

$\Rightarrow$ parity violated

VŠCHT PRAHA

Chirality (and helicity)

chiral

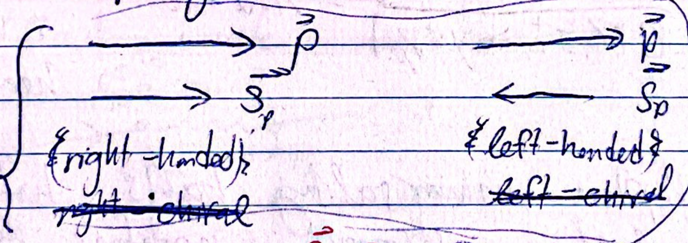
// handedness of particle (frame invariant) χ

- weak force interacts only with left-handed states (right-handed antiparticle)

$\Rightarrow$ parity violation

$e^-_L,\ e^+_R$

$q_L,\ \bar{q}_R$



vs. Helicity $h = \dfrac{\vec{S}\cdot\vec{p}}{|\vec{S}||\vec{p}|} = \pm 1$ interesting, fully polarised particle but can be anything in between

$\downarrow$

kinematic property

$\Rightarrow$ frame dependent

if $h = +1$ $\Rightarrow$ right-handed ($\vec{p}$ and $\vec{S}$ aligned)

if $h = -1$ $\Rightarrow$ left-handed

mass of the particle couples h and χ: $|\chi_L\rangle \simeq \left(1-\frac{M}{2E}\right)|h_L\rangle + \left(\frac{M}{2E}\right)|h_R\rangle$

- if particle has mass, it can change sign based on frame
- if particle is massless, its helicity is fixed and then $h = \chi$ for $m=0$

$\hookrightarrow |\chi_L\rangle = |h_L\rangle$

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$\triangle$ $\pi^+ \to l^+ \nu_e$ $h=-1 \left\{ \begin{matrix} p \leftarrow \\ s \rightarrow \end{matrix} \right. \overset{e^+}{\longleftarrow} \overset{\pi^+}{\bullet} \overset{\nu_e}{\longrightarrow} \left. \begin{matrix} p \\ s \end{matrix} \right\} h=-1$

$\overset{\shortparallel}{u\bar{d}}$, $S=0, L=0$

$\left.\begin{matrix} \cdot\ \vec{S}_{e^+} = -\vec{S}_{\nu_e} \\ \cdot\ \vec{p}_{e^+} = -\vec{p}_{\nu_e} \end{matrix}\right\} \Rightarrow \boxed{h_{e^+} = h_\nu}$

but $\left.\boxed{\begin{matrix} \chi_{e^+} = +1 \\ \chi_{\nu_e} = -1 \end{matrix}} \begin{matrix} = \text{right-chiral} \\ = \text{left-chiral} \end{matrix}\right\} \Rightarrow \boxed{\chi_{e^+} = -\chi_{\nu_e}}$

$\rightsquigarrow \Rightarrow$ for neutrinos (approx. massless)

$\Rightarrow \chi_\nu \approx h_\nu$

$\Rightarrow$ $h_e = h_\nu \approx \chi_\nu = -\chi_{e^+}$

$\Rightarrow$ $\underline{h_e = -\chi_e}$?! $\Rightarrow$ e.g. right-~~[illegible]~~chiral and left-handed

$\Rightarrow$ $\boxed{\chi_e = +1 \text{ and } h_e = -1}$

$\Rightarrow$ chirality and helicity are opposite for a massive lepton

For μ^+ easier to satisfy $\chi_\mu = -h_\mu$ but for e^+ difficult because of low mass

$$\frac{BF(\pi^+ \to e^+\nu_e)}{BF(\pi^+ \to \mu^+\nu_\mu)} = 1.2 \times 10^{-4}$$

$\Rightarrow$ electron decay mode is heavily suppressed

$\Rightarrow$ lepton universality holds but mass of lepton strongly modifies Branching Fraction.

$\hookrightarrow$ very important effect for spinless 2-particle weak decays

- also for quarks: left-handed quarks
  right-handed antiquarks
- expected: only transitions within generations
  but actually not e.g. $\Lambda(uds) \xrightarrow{s \to u} p(uud)$
- size of weak charge same for all quarks
  $\Rightarrow$ quark universality

Cabibbo mechanism – Higgs and W have different quark eigenstates ⇒ mixing for W

• hypothesis: parametrise quarks as linear combinations:

$$\langle u|d'\rangle = \underbrace{\cos\theta_c}_{g_{ud}} |d\rangle + \underbrace{\sin\theta_c}_{g_{us}} |s\rangle$$

$$\langle c|s'\rangle = \underbrace{-\sin\theta_c}_{g_{cd}} |d\rangle + \underbrace{\cos\theta_c}_{g_{cs}} |s\rangle$$

new quarks

~5% correction

Cabbibo angle θ_c (small ⇒ cos dominates) → 10/15°

⇒ main decay within generations but small correction across generation (sin term)

$g_{ud} = \cos\theta_c \, g_w$
$g_{us} = g_w \sin\theta_c$
$g_{cd} = -g_w \sin\theta_c$
$g_{cs} = g_w \cos\theta_c$

↓

$g_{ud} = g_w \cos\theta_c$ and $g_{us} = g_w \sin\theta_c$

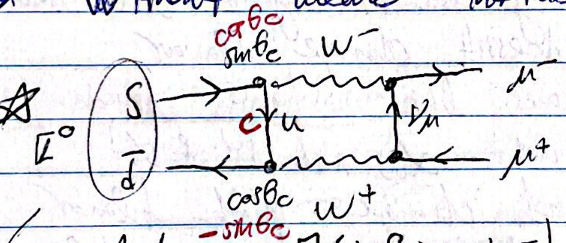
coupling of u/d transition $u \leftrightarrow d$

coupling of u/s $u \leftrightarrow s$ transition

☆ $$\frac{\Gamma(K^- \to \mu^- \bar{\nu}_\mu)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \propto \frac{g_{us}^2}{g_{ud}^2} = \tan^2\theta_c$$

$0.2313^2 \Rightarrow \theta_c \approx 13°$

☆ Without weak interactions, many more stable states.

☆ 

prediction: $\Gamma(K^0 \to \mu^+\mu^-) \propto \cos\theta_c \sin\theta_c \approx 10^{-3}$

but no branching fraction observed

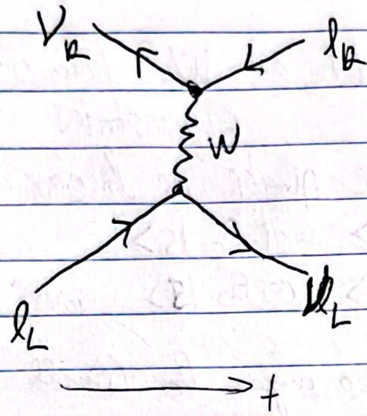
⇒ GIP mechanism: also c possible

but $|s'\rangle$ modifies sc coupling

→ $g_{ud}' = g_{ud} + g_{us}$

both options ⇒ $\Gamma(K^0 \to \mu^+\mu^-) = \cos\theta_c \sin\theta_c - \cos\theta_c \sin\theta_c = 0$

(currently measured 0 at 10^{-10} precision)

☆ 

- right-chiral
- left-chiral

⇒ violation parity ↓ **maximal**

- strong, elmag mediated by vector current (V)
- weak → lefthanded ⇒ mediated by vector - axial vector current (V-A)

minus, vector

$\hat{P}$: flips sign / stays same

⇒ we need both to violate parity (one symmetric, one asymmetric)

Z and quark mixing (using W mixing)

$|d'd'\rangle = \cos^2\theta_c |dd\rangle + \sin^2\theta_c |ss\rangle + \sin\theta_c\cos\theta_c (|ds\rangle + |sd\rangle)$

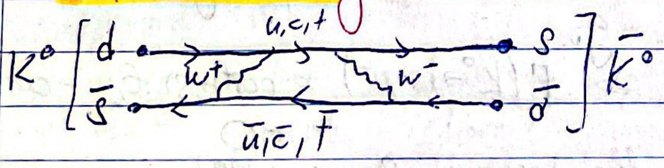
$|s's'\rangle = \cos^2\theta_c |ss\rangle + \sin^2\theta_c |dd\rangle - \sin\theta_c\cos\theta_c (|ds\rangle + |sd\rangle)$

$= 0$

adding up: $|d'd'\rangle + |s's'\rangle = |dd\rangle + |ss\rangle + 0$

⇒ keeps only flavour diagonal contributions
= doesn't change flavour
⇒ Z behaves like a heavy photon
(just with α_W instead of α)
⇒ Z cannot change quark flavour
⇒ even in weak force, we can only change flavours diagonally ($s \leftrightarrow d$ is not allowed, while $s \leftrightarrow u$ is allowed)

Meson mixing



by convention

$\star$ $C|K^0\rangle = -|\bar{K}^0\rangle$ $\qquad$ $P|K^0\rangle = -|K^0\rangle$

$C|\bar{K}^0\rangle = -|K^0\rangle$ $\qquad$ $P|\bar{K}^0\rangle = -|\bar{K}^0\rangle$

Now apply CP: $CP|K^0\rangle = |\bar{K}^0\rangle$

$CP|\bar{K}^0\rangle = |K^0\rangle$

$\Rightarrow$ define a set of CP eigenstates by LC of $|K^0\rangle$ and $|\bar{K}^0\rangle$

$= |K_1^0\rangle, |K_2^0\rangle$

then $CP|K_1^0\rangle = +|K_1^0\rangle \propto |K^0\rangle + |\bar{K}^0\rangle$

$CP|K_2^0\rangle = -|K_2^0\rangle \propto -(|K^0\rangle - |\bar{K}^0\rangle)$

normalisation: $\frac{1}{\sqrt{2}}$

Sometimes $K_S^0 \approx K_1^0$, $K_L^0 \approx K_2^0$ given LC's

CP states

$K_{1,2}$

$\Rightarrow$ when there is a decay, we do not see K^0 nor $\bar{K}^0$, we observe K_1^0 or K_2^0 – the CP eigenstates

$$\begin{cases} K_1^0 \to \pi\pi, & CP = +1 \\ K_2^0 \to \pi\pi\pi, & CP = -1 \end{cases}$$

$\Rightarrow$ more energy release, more likely decay, smaller lifetime, larger decay width

CP violation in K-decay

- make K_L^0 beam
- search for $K_1^0 \to \pi^+\pi^-$
  - $\Rightarrow$ happened and found! $\Rightarrow$ CP violated!

CP violation in mixing

- observe decay of K^0 or $\bar{K}^0$ state

$K^0 \to \pi^- e^+ \nu_e$

$\bar{K}^0 \to \pi^+ e^- \bar{\nu}_e$

- for small CP violation

$$|K_S^0\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(|K_1^0\rangle + \varepsilon|K_2^0\rangle\right)$$

$$|K_L^0\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(\varepsilon|K_1^0\rangle + |K_2^0\rangle\right)$$

$\Rightarrow$ express K^0 and $\bar{K}^0$ states and work out asymmetry to find $2\mathrm{Re}(\varepsilon) = \frac{N_+ - N_-}{N_+ + N_-}$

Adding third generation ...

~Cabbibo but incl. all 3 generations

unitary — CKM matrix

learn

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

for Cabbibo: $V \sim \sin/\cos\theta_c$

here: 3 parameters, and 1 imaginary → phase shift

$\boxed{\text{Without 3rd generation, we cannot violate CP and have matter/antimatter asymmetry}}$

amplitudes:
$$\begin{bmatrix} 0.974 & 0.225 & 0.003 \\ 0.225 & 0.973 & 0.041 \\ 0.009 & 0.040 & 0.999 \end{bmatrix} \rightsquigarrow \text{square} \to \text{even larger suppression}$$

↳ most likely to couple within generations

↳ further couplings are weaker

↳ .

$$\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

	d	s	b
u	1	2	4
c	2	1	3
t	4	3	1

↳ order of strength of coupling

1^{st}–2^{nd} stronger than 2^{nd}–3^{rd} (closer in mass)

$0.225 = \lambda$ = order of elements (suppression)

$0.8132 = A$ = additional corrections

$0.1607 = \rho$ = — " —

$0.3565 = \eta$ = imaginary component (phase shift)

↳ CP violation

↳ changes sign under CP transformation

$\boxed{\text{Problem: CKM contribution not large enough to explain matter/antimatter symmetry} \sim \text{CP violation}}$

in B-decay: CP violating contribution dominates (CP conservation is small)

Neutrinos

- chirality : ν_L, $\bar{\nu}_R$ - via weak force
  - or maybe $\bar{\nu}_R = \nu_R$ (ν its own antiparticle)
  - (ν_R, $\bar{\nu}_L$ don't interact with anything / → except for possibly by gravity (not detected yet) ⇒ interaction numbers 0
- only via weak force
- ~~if no mass only left helicity~~
- in SM, no masses
  - → no direct observation of Higgs interaction

Neutrino sources

Natural

- most common — solar neutrinos (ν_e) → from sun
- cosmic rays (atmospheric neutrinos) (all)
  - particle from space interacts in atmosphere ⇒ creates ν
- cosmic neutrinos (all)
  - neutrinos coming from far away space ↳ can be used to see further back in past in the Big Bang (better than CMB radiation)

Artificial

- nuclear reactor ($\bar{\nu}_e$)
- particle beams (all)

Neutrino detection

- less common → need lots of material and hidden underground / under ice to remove other particle background

Homestake experiment

- solar neutrinos
- $\nu_e + Cl \rightarrow Ar^+ + e^-$ (inverse β decay)
- only observed 33% of expected solar neutrino rate
  - ⇝ only sensitive to electron neutrinos!
  - measure $\boxed{\nu_e \to \nu_e}$

Super Kamiokande

- water tank (huge) → Chenenkov radiation
  - → photomultiplier tubes
- measure flux of ν_μ vs. distance travelled
  - ⇒ at high $\frac{L}{E}$ (distance/energy), many ν_μ disappear
  - but neutrinos oscillate in flavour ⇒ we don't keep ν_μ
  - ↳ measure $\boxed{\nu_\mu \to \nu_\mu}$

Neutrino mixing

- if neutrinos have mass ⇒ different interaction and mass eigenstates

$$\nu_\alpha = \nu_i \cos\theta_{ij} + \nu_j \sin\theta_{ij}$$
$$\nu_\beta = -\nu_i \sin\theta_{ij} + \nu_j \cos\theta_{ij}$$

} similarly to Cabibbo

interaction eigenstates

{ mass eigenstates $\nu_{i/j}$ different masses ⇒ energies

- for quarks : first mass eigenstates → interaction states different (mixed mass states)

vs. neutrinos: first interaction eigenstates → ⇒ mass eigenstates as LC of interaction states

$\Psi(t) \propto e^{-iEt}$ ⇒ time evolution dependent on energy

but $E_i \neq E_j$ ⇒ the mass states propagate differently in time ⇒ ratios ~~of~~ of ν_i and ν_j change in time

⇒ pure ν_α state propagates as

$$|\nu_\alpha\rangle(t) = a_i(t)\,|\nu_i\rangle \cos\theta_{ij} + a_j(t)\,|\nu_j\rangle \sin\theta_{ij}$$

⇒ after time $t \neq 0$, ν_β contribution in $\nu_\alpha(t)$

⇒ neutrino oscillation

$$|\nu_\alpha\rangle(t) = A(t)\,|\nu_\alpha\rangle + B(t)\,|\nu_\beta\rangle$$

$$\Rightarrow \quad B(t) = \sin\theta_{ij}\cos\theta_{ij}\,(a_j(t) - a_i(t))$$

Neutrinos can change flavour while travelling → possible only if (at least some) neutrinos have mass and mix

$v_\nu \sim c$ ⇒ express as function of distance travelled L (instead of time)

$$\Rightarrow \boxed{P(\nu_\alpha \to \nu_\beta) = \sin^2\theta_{ij}\,\sin[(E_j - E_i)t] = \sin^2\theta_{ij}\,\sin\left[\frac{m_j^2 - m_i^2}{4E}L\right]}$$

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mass eigenstates

PMNS matrix

not hierarchical! (vs. quarks)

close to anarchical

interaction eigenstates

	ν_1	ν_2	ν_3
ν_e	1	3	5
ν_μ	4	3	2
ν_τ	4	3	2

$\sin^2\theta_{12} = 0.304$

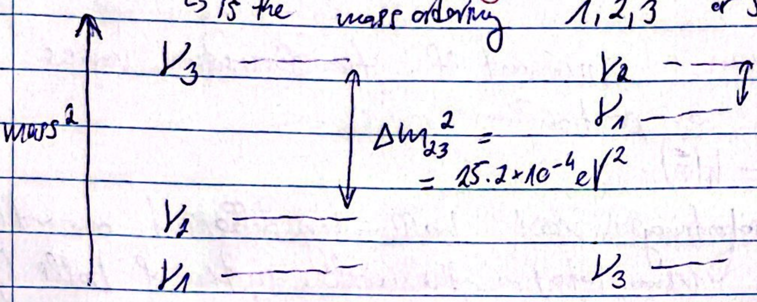
$\sin^2\theta_{23} = 0.573$

$\sin^2\theta_{13} = 0.022$

↳ larger mixing then for quarks

Neutrino mass hierarchy?

↳ is the mass ordering 1,2,3 or something else? We only know differences



Normal? Inverted?

Introducing a mass term

- for quarks and charged leptons:

Dirac $\Rightarrow m\bar{\Psi}\Psi = m(\bar{\Psi}_L + \bar{\Psi}_R)(\Psi_L + \Psi_R) = m(\bar{\Psi}_L\Psi_R + \bar{\Psi}_R\Psi_L)$

left + right handed components

diagonal terms drop out

- but for neutrino we cannot observe Ψ_R and $\bar{\Psi}_L$
  ⇝ we'd have to introduce them to have a mass term

⇝ can be solved by Majorana neutrinos ($\bar{\Psi}_{R,L} = \Psi_{R,L}$) ⇝ they are its own antiparticle

Number of neutrinos

- measuring total decay width to test possible contribution from 4th neutrino (generation)
  ~~but~~ found: $\# \nu = 2.984 \pm 0.008$
  but excluding "neutrinos" with $M_\nu > \frac{M_Z}{2} \sim 45\,GeV$ (Z boson)
- Z boson couples to neutrinos directly (even to itself)
  → usually only to particle/antiparticle pair)

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Z boson motivation

- predicted before discovery
- higher order interaction diagrams $e^+e^- \to \mu^+\mu^-$
  based on only W don't converge ($\int \to \infty$)
  $\Rightarrow$ neutral particles ~~converge~~ fix it
  (subtract off the infinities)
- observed as neutral particle (boson) coupling to ν

Local symmetries

(?) Is the system invariant if transformation varies depending on position?

$f(\vec{r}) \to f(\vec{r}) + h(\vec{r})$

$\rightsquigarrow$ e.g. rotating each ball by different amount (local)
vs. rotation the whole system of balls (global)

↳ similar to Gauge symmetries in elmag.

$$\begin{cases} \vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t} \\ \vec{B} = \nabla \times \vec{A} \end{cases}$$

$\phi, \vec{A}$ – not observable
$\rightsquigarrow$ can be modified while keeping "real" stuff $(\vec{E}, \vec{B})$ same

$\Rightarrow$ invariant under transformations:

$$\boxed{\begin{aligned} \phi &\to \phi + \frac{\partial f}{\partial t} \\ \vec{A} &\to \vec{A} - \vec{\nabla} f \end{aligned}}$$

Lorentz gauge condition
$$\frac{\partial \phi}{\partial t} + \nabla \cdot \vec{A} = 0$$

"gauge transformation" because modifies potentials but keeps observables same

$\Rightarrow$ any theory expressed in terms of potentials should be gauge invariant

$\rightsquigarrow$ Gauge symmetry in QED

$$\psi(\vec{r},t) \to \psi'(\vec{r},t) = e^{iqf(\vec{r},t)}\,\psi(\vec{r},t)$$

↳ wavefunction can be modified by a phase (drops out when squaring) ~~$\Rightarrow$~~ local (gauge) transform.

↳ + Lorentz gauge transform $\Rightarrow$ eq. of motion for potentials

Klein-Gordon eq's:
$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\phi = 0 \quad +m\phi \text{ if } m_\gamma \neq 0$$
$$-\,\text{"}\,-\ \vec{A}$$

same as massless spin-1 particle $\Rightarrow$ photons

Massive γ would break

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Gauge symmetries generate interactions

what can I change in my potential?

→ change it → require observables to be unchanged → profit!

↳ apply to Dirac eq. $\Psi(\vec{r},t) \to \Psi'(\vec{r},t) = e^{iqf(\vec{r},t)}\Psi(\vec{r},t)$

$$\Rightarrow i\left(\frac{\partial}{\partial t} + iq\frac{\partial f}{\partial t}\right)\Psi' = -i\vec{\alpha}\left(\nabla + iq\nabla f\right)\Psi' + \beta m\Psi'$$

extra terms

transformation?

$\frac{\partial}{\partial t} \to \frac{\partial}{\partial t} + iq\phi$

$\nabla \to \nabla - iq\vec{A}$

⇒ similar behaviour to electromagnetism

⇒ electromagnetism naturally arises

⇒ looking at possible gauge symmetries and exploring options, yields interactions

Electroweak interaction

- gauge transform for weak force ⇒ terms with same coupling for $W^{+/-}$ and W^0

charged currents / neutral current

but neutral was found to have a smaller coupling then $W^{+/-}$

⇒ solution: add additional field B^0 which mixes with W^0 field to yield Z^0 and γ

$$\begin{cases} \gamma = B^0\cos\theta_W + W^0\sin\theta_W \\ Z = -B^0\sin\theta_W + W^0\cos\theta_W \end{cases}$$

+ gauge + neutrinos don't interact with γ

$$\Rightarrow \frac{e}{2\sqrt{2\varepsilon_0}} = g_W\sin\theta_W = g_Z\cos\theta_W \quad , \quad \cos\theta_W = \frac{M_W}{M_Z}$$

g_Z = coupling of Z

⇒ mass predictions for W and Z before their discovery

Gauge transformations in SM

$$\underset{\text{QED}}{U(1)_Y} \otimes \underset{\text{QFD}}{SU(2)_L} \otimes \underset{\text{QCD}}{SU(3)_C}$$

$U(1)_Y$: B^0 ← ; rotations in hypercharge space; 1 DOF; γ, Z

$SU(2)_L$: $W^{+/-/0}$; rotations in weak isospin space; 3 DOF; $W^{+/-}$, Z^0

$SU(3)_C$: rotations in color space; 8 DOF; 8 gluons

DOF = degree of freedom

Issue: $W^\pm, Z^0$ masses are not gauge invariant ⇝ Higgs to fix

Higgs field

Boson

- $M_H = 125\,GeV$
- Spin = 0 → scalar field

- gives elementary particles mass (quarks, leptons $\to \nu$?, W, Z)
  Higgs allows us to write down a mass term without breaking gauge invariance
- solves some theoretical issues

Detection

- CMS, ATLAS at LHC

Mass terms in Lagrangians

- $\mathcal{L}$ for particle w/o interactions - free particle with mass m

$$\mathcal{L} = \frac{1}{2}\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\phi^\dagger\phi - \frac{1}{2}m^2\,\phi^\dagger\phi = T(\phi) - V(\phi)$$

↳ quadratic terms in potential → mass ($m\phi^2$)

(higher order terms in potential are for interactions)

- Higgs adds mass terms dynamically
- coupling of Higgs to particle $\propto m^2_{particle}$
- Yukawa couplings to Higgs are dimensionless

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\Psi}\not{D}\Psi + \bar{\Psi}_i\, y_{ij}\,\Psi_j\,\phi - V(\phi) + |D_\mu\phi|^2$$

gives mass to fermions — Yukawa couplings (dimensionless)

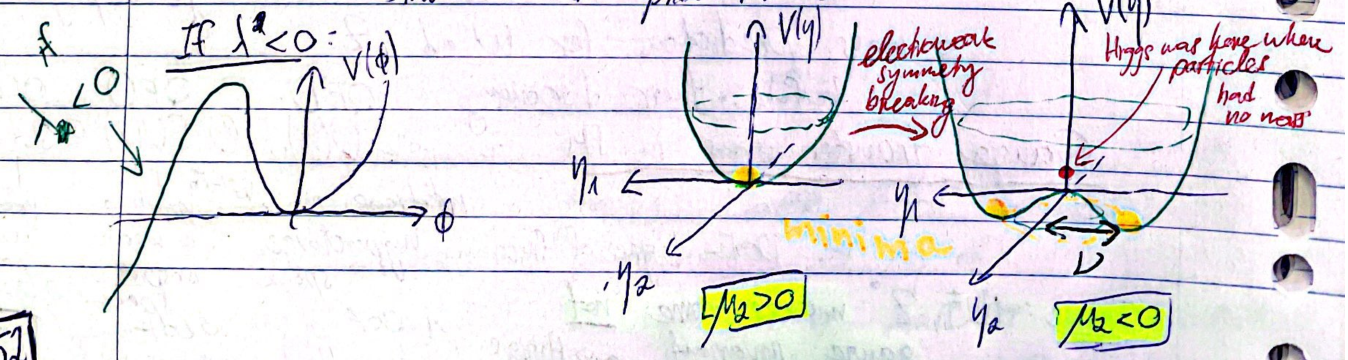
ϕ = Higgs field

$|D_\mu\phi|^2$: propagator of Higgs field

Higgs potential: $V(\phi) = \mu^2\phi^\dagger\phi + \frac{\lambda}{4}(\phi^\dagger\phi)^2$ (change in time and space), $\lambda > 0$ (or λ^2?)

Higgs field = complex scalar field $\phi(\vec{r},t) = \eta_1(\vec{r},t) + i\,\eta_2(\vec{r},t)$

↳ invariant under phase rotations



- vacuum state given by minimising potential energy
  - if $\mu^2 > 0$, minimum for $|h(\vec{r},t)| = 0$ (no energy in H-field)
  - if $\mu^2 < 0$, minimum for $|h(\vec{r},t)| = \sqrt{-\frac{\mu^2}{2\lambda}}\, e^{i\theta}$ phase

    but phase doesn't matter for observables

$$\Rightarrow \boxed{v = \sqrt{-\frac{\mu^2}{2\lambda}}} \approx 242\,GeV \quad \text{actual minimum of potential}$$

~~potential~~ = spontaneous symmetry breaking
(Higgs ended up in particular place on a circle but could have gone anywhere)

happened some time after Big Bang when the potential drop to the $\mu < 0$ case (think cosmology)

happened when particles got mass

$\Rightarrow$ expansion around minimum

$$\mathcal{L} = \left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) h^\dagger h - \lambda v^2 h^\dagger h - O(h^3)$$

$$\Rightarrow m_h = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2} \quad = \text{mass of scalar field}$$

$\rightsquigarrow$ massive spin-0 particle

for <u>weak force</u>: (electroweak unification)

$$M_W = \frac{1}{2} v g_W$$

g_W ← weak coupling

$$g_{Hff} = \sqrt{2}\, g_W \frac{m_f}{m_W}$$

g_{Hff}: coupling of fermions to Higgs

- λ is a free parameter $\Rightarrow$ m_{Higgs} is not predetermined

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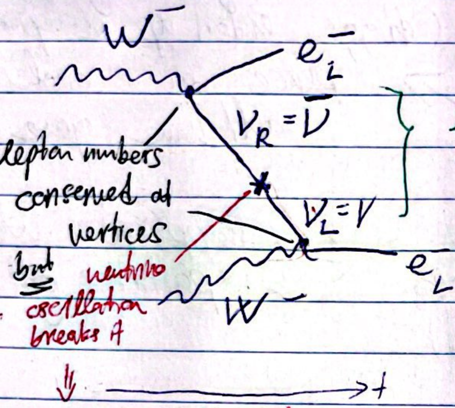
- Fermion masses are generated from Yukawa term (for quarks and charged leptons)
- if neutrinos have **Dirac** masses ($m \nu_L \bar{\nu}_R + m \nu_R \bar{\nu}_L$)
  $\rightsquigarrow$ no way to observe $\nu_R, \bar{\nu}_L$

L, R = chirality

- vs. Majorana masses ($m \nu_L \nu_R$)
  $\Rightarrow$ directly breaks lepton flavor conservation
- if $m_\nu = 0$, no way to distinguish between scenarios

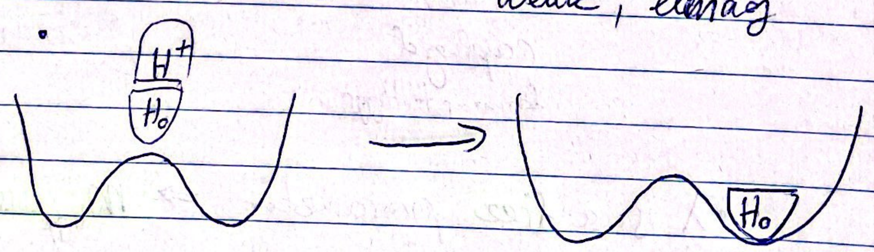


initial: $L = 0$ $(W + W)$
final: $L = 2$ $(2e^-)$

- Majorana mass term (i.e. neutrinos are Majorana) would allow double-beta decay w/o ν = neutrino turns into its own antiparticle and only $2e^- / 2e^+$ are emitted

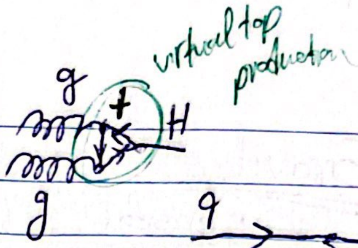
Symmetry Breaking

- $W^{+/-/0}, B^0 \longrightarrow W^{+/-}, Z^0, \gamma$
  electroweak $\longmapsto$ weak, elemag
-

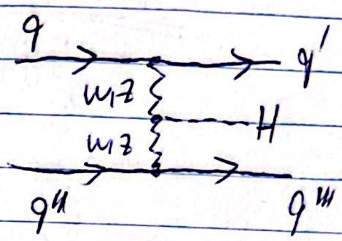


Higgs production

high coupling to top — • gluon fusion



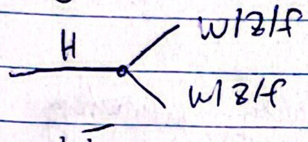
• vector boson fusion



high coupling to W/Z — • associated production

decay

• many decay modes – depends on mass



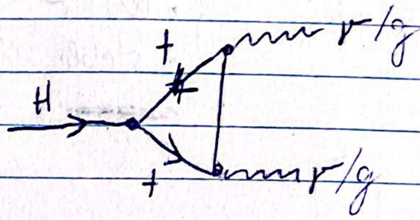
$H \to b\bar{b}$ – 57.7%

$H \to WW^*$ – 21.5%

$H \to \tau\tau$ – 6.3%

$H \to ZZ^*$ – 2.6%

real / virtual – quickly decays before end of interaction
– cannot be backtracked from ~~its mass~~ the energy of the products (Z^* has negative kinetic energy)



Beyond Standard Model

Problems with SM? — unexplained features of theory (why?)

- dark vacuum energy
- inconsistent measurements (measurement vs theory)
  - B quark
  - g-2 measurements
- gravity & SM
- unification of forces (strong + elweak)
- (anti)matter asymmetry
- neutrino masses (mixing)
- dark matter

Gravity

- graviton (spin 2) ⇝ un-renormalizable
  tensor current
  if included basically

Dark Energy Λ (69%)

- simple calc. of vacuum energy of space in SM is off by 120 orders of magnitude

Dark Matter (26%)

- WIMP - weakly interacting massive particles
- stable

~~candidate~~ ⇝ neutrinos? No, too light
⇝ modified gravity?
⇝ primordial black hole?

Matter - Antimatter asymmetry

- at high temp, in thermal eq. (BB)
  ⇒ production of equal amounts of matter/antimatter

⇝ then cooling sus
⇝ now #matter > #antimatter

- SM gives prediction for amount of matter/antimatter asymmetry : 10 orders of magnitude off from measurement !!

- Sakharov conditions for asymmetry
  - violation of baryon number B
  - violation of C and CP symmetries
  - interactions out of thermal equilibrium

what SM predicts is not enough for these
⇒ need new sources of B and CP violation

Measurement inconsistencies

- no commonly accepted deviations
- precision measurements (muon $g-2$)
  - get more precise numbers

Theory issues

- Hierarchy problem – large quantum corrections to H_{mass} → higher order diagrams
  - some corrections are very large but H_{mass} cancels out
- No unification of forces – no widely accepted
- Flavour puzzle
  - masses and CKM parameters depend on gen.
  - Why 3 gens?
  - Why neutrinos different

- expect 1 force at higher energies which is broken into 3 at lower energies
- expected at $\sim 10^{15}$ GeV but in SM interaction couplings do not converge at same energy
- unified forces couples baryons to leptons
  - → baryon number ~~con~~ violation
  - ⇒ proton could decay into leptons with lifetime 10^{30} yrs
  - Not observed ⇒ unification doesn't work

SOLUTIONS

- Classical approach : think of model to solve certain problems
- Modern approach : combine measurements in consistent framework, search for discrepancies w/ predictions

Neutrinos

- potential for Beyond SM because of mixing, CP violation ...
- Seesaw mechanism for small neutrino masses
- 4th non-interacting neutrinos

$$M_\nu = \begin{pmatrix} 0 & m_D \\ m_D & m_M \end{pmatrix} \quad m_D \sim m_{EW} \sim m_W, m_Z \sim 100 \text{ GeV}$$

Dirac+Majorana

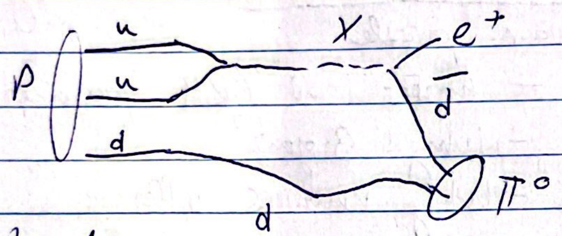
$$m_M \sim m_{GUT} \sim m_X \sim 10^{15} \text{ GeV}$$

→ neutrino ~~masses~~ eigenstate masses: m_M (heavy ν), $\frac{m_D^2}{m_M}$ (small neutrino mass)

• Finding heavy neutrinos (exact masses unknown)
~> study mass range vs. coupling strength
majorana => can produce two charged leptons with same sign => look for this

• Looking for sterile neutrinos : $\nu_R, \bar{\nu}_L$ (no weak interact.
– mix with other neutrinos ~> disappearance

GUT



• leptoquarks

Supersymmetry
– using symmetries → make particles but with less spin
– each particle gets a super-partner (Squark, slepton, ... Wino, Bino, Higgsino
boson (integer spin) → fermion (half integer spin)

$S_{\tilde{q}} = 0$ and $S_{\tilde{\nu}} = 1/2$

$S_{\tilde{p}} = 0$
$S_{\text{bosons}} = 1/2$ } – 1/2
$S_{\tilde{H}} = 1/2$

– virtual contributions from particle and superparticle almost exactly cancel
– lightest superparticle (neutralino*)
↳ its own conserved quantum number => cannot decay because lightest
↳ candidate for DM
but not found yet

* diracg neutral
Linear combination of $\tilde{g}, \tilde{Z}, \tilde{W}, \tilde{H}$

– too many parameters X
– no signs found X

– would solve Higgs corrections ✓
DM ✓
unification of forces ✓